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NONLINEAR TRANSMISSION CHARACTERISTICS OF THE PLASMA SHEATH

Robert J. Papa

ERRATA

1. Page 100, omitted at the time of printing, is as follows:

68. P. Molmud, "Altering the Electron Density in the D-region, A New Tool for Ionospheric D-layer Research," Space Tech. Labs., Scientific Report No. 3 Contract AF19(604)-8844, AFCRL-62-533 (Jul 1962).

69. M. M. Klein, H. D. Greyber, J. I. F. King and K. A. Brueckner, "Interaction of a Nonuniform Plasma with Microwave Radiation," in Electromagnetic Effects of Re-entry ed. by W. Rotman and G. Meltz, Pergamon Press (1961).

70. K. A. Graf and M. P. Bachynski, "Transmission and Reflection of Electromagnetic Waves at a Plasma Boundary For Arbitrary Angles of Incidence," Can. Jour. of Phys. 39 (1961).

71. R. Papert, and G. Plato, "Study of Magneto-Ionic Modes with Application to Telemetry," (CONFIDENTIAL REPORT) Convair Physics Section Report Z Ph - 081 (1961).

72. L. S. Taylor, IRE Trans. on Antennas Prop, AP-9, 483 (1961); AP-9, 582 (1961).

73. S. J. Buchsbaum, "Interaction of EM Radiation with a High Density Plasma," Ph. D. Thesis, MIT (1957).

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75. S. A. Schelkunoff, "Remarks Concerning Wave Propagation in Stratified Media," p. 181, "The Theory of Electromagnetic Waves" Interscience (1951).

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77. F. A. Albini and R. G. Jahn, "Reflection and Transmission of Electromagnetic Waves at Electron Density Gradients," Jour. Appl. Physics, Vol 32, No. 1 (Jan 1961).

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81. J. I. F. King and Gray, "Heat Shield Response of an Irradiated Plasma," G. E. Space Sciences Lab Report No. 8 AF30(602)-1968 (Oct 1960).

82. J. I. F. King, "R. F. Heating of an Inhomogeneous Plasma," G. E. Space Science Lab., Report No. 7, ARDC TR-60-219, Contract AF30(602)-1968.

83. M. S. Sodha, "Heating of an Ionized Gas Sheath by Microwaves," Armour Research Foundation of Illinois, Inst. of Tech. Project Suggestion No. 60-107AX (May 1960).

84. E. M. Dewan "Generalizations of the Saha Equation," AFCRL Report No. 42 (May 1961).

2. On catalog cards, change date from 'February 1963' to 'May 1963.'

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Research Report

Nonlinear Transmission Characteristics of the Plasma Sheath

ROBERT J. PAPA

MICROWAVE PHYSICS LABORATORY PROJECT 4642
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES, OFFICE OF AEROSPACE RESEARCH, UNITED STATES AIR FORCE

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Abstract

A model is constructed of a nonlinear, one-dimensional inhomogeneous multicomponent plasma slab which has the characteristics of the plasma sheath surrounding a typical hypersonic re-entry vehicle at 200,000 ft while traveling at about 18,000 ft/second. A plane wave incident upon the slab at arbitrary angle can induce changes in electron temperature without affecting the neutral gas temperature. The variations of electron temperature produce changes in collision frequency and the rate coefficients describing the various electron production and loss mechanisms. Such changes cause alterations in the effective dielectric constant of the medium. On the basis of a model of a plane-layered medium composed of a stack of linear homogeneous sheets, the reflection and transmission coefficients of the nonlinear slab may be computed by a step-by-step numerical integration of Maxwell's equations expressed in the form of difference equations.

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Contents (Contd)

Nonlinear Transmission Characteristics of the Plasma Sheath

1. INTRODUCTION

A hypersonic re-entry vehicle is enveloped by a shock-induced sheath of ionized gas. This re-entry plasma sheath can greatly influence electromagnetic communications frequencies in the radio and microwave range. Many calculations have been made on the reflection and transmission properties of the plasma sheath surrounding a hypersonic re-entry vehicle. Perhaps the most straightforward analysis, which can yield useful information on signal degradation, is based upon the model consisting of a plane wave incident on a dielectric slab. The electromagnetic properties of the re-entry plasma sheath are characterized by an equivalent dielectric slab whose complex dielectric constant and thickness simulate the actual plasma sheath. However this type of analysis is not capable of predicting the dominant characteristics of the radiation patterns of plasma-coated slot antennas located on a re-entry vehicle surface.

If communications between the vehicle and ground are effected by means of slots or horns flush-mounted on the missile surface, a suitable model describing the antenna system consists of a slot-excited plasma slab covering a ground plane. The prominent features of the radiation patterns of such a structure may be obtained by evaluating the integral representations for the field components by the method of steepest descent. Tamir and Oliner[1] have demonstrated that the near field of a slot-excited plasma slab is represented mainly by contributions from complex waves.

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For frequencies below the plasma frequency, the complex waves are of the spectral type which do not transport energy but represent stored energy. At frequencies above the plasma frequency, the complex waves are of the leaky-wave type and may be strongly excited for certain slab thicknesses. The shape of the far-field radiation pattern may be obtained by integrating the near field over a Kirchoff-Huygens surface. The near-field components are found by evaluating the integral representations of the field by the calculus of residues. For frequencies greater than the plasma frequency, the radiation pattern of a slot-excited grounded plasma slab exhibits peaks at the critical angle. Shore and Meltz[2] have investigated the far-field components using Fourier analysis and the stationary phase procedure for the case of a slot in a ground plane covered by an anisotropic lossy plasma layer.

These methods of analysis usually assume a uniform plasma layer, although inhomogenieties in one dimension may be taken into account by using a plane-layered model. Felsen and Marcuvitz[3] have pointed out that an electromagnetic field problem with prescribed sources may be reduced by suitable modal procedures to a network problem. The equivalent network problem may then be solved for a typical mode and the field solution is then obtained by modal synthesis. The problem of the modal analysis of a plane-layered medium may be transformed into an equivalent network problem of a transmission line consisting of uniform sections, where the voltage and current at the discontinuities in the line have the same boundary conditions as the E and H fields have at the discontinuities in the plane-layered medium.

All of these approaches to the problem of electromagnetic transmission through the re-entry plasma sheath do not consider the effect of perturbations induced in the plasma medium by the presence of an electromagnetic wave. If the electromagnetic propagation characteristics of the plasma medium are described by a complex dielectric constant K, then this parameter may be related to the electron density N_e and electron-neutral collision frequency ν:

$$K = K_r + jK_i$$

$$= 1 - \frac{(\omega_p/\omega)^2}{1 + (\nu/\omega)^2} - j\frac{(\nu/\omega)(\omega_p/\omega)^2}{1 + (\nu/\omega)^2} \quad . \tag{1}$$

For relatively high field strengths, it is expected that the degree of ionization and the electron-neutral particle collision frequency will change, thereby altering the electromagnetic transmission characteristics of the plasma medium. However, any model for describing such a nonlinear interaction between high-power electromagnetic radiation and the plasma medium will have to include a detailed picture of the interaction of the local fields inside the medium with the plasma constituents.

A detailed picture of the various microscopic processes, such as the local field-particle interactions and the particle-particle interactions, give rise to a medium that may be characterized on a macroscopic level by a complex dielectric constant that is dependent upon the local field distribution in the plasma medium. This type of medium does not lend itself to modal analysis, since the representation of an arbitrary field in the medium by a Fourier integral over the wave numbers depends upon the validity of the principle of superposition. Each mode is assumed to propagate with a phase velocity that depends upon the dielectric constant of the medium, and the dielectric constant is assumed to be independent of the presence of other modes. This condition is no longer satisfied in a medium whose dielectric constant depends upon the local field distribution in the medium. For this reason a nonlinear model was constructed consisting of a plane wave of arbitrary angle of polarization incident at an arbitrary angle upon a plane-layered plasma slab. The slab consists of all the constituents normally found behind a high-temperature shock (5000°K). The thermodynamic and gas kinetic parameters were selected to coincide with conditions found behind a shock of a representative blunt-nosed vehicle traveling at 18,000 ft/sec at an altitude of 200,000 feet.

The total reflection and transmission coefficients of a plane-layered plasma slab having the same characteristics as the sheath surrounding a typical re-entry vehicle may be computed for an incident plane wave consisting of a single frequency component. The nonlinear response of the plasma medium to the perturbing electromagnetic wave is found for the steady-state case, corresponding to an incident plane wave whose frequency is much greater than the frequency for energy transfer between the electron gas and the neutral-particle gas. The condition for a steady-state nonlinear response of a plasma to a perturbing electromagnetic wave may be written

$$\omega \gg G\nu ,$$

where
- ω = frequency of impressed field
- G = relative fractional energy loss of an electron in colliding with a neutral particle ($G = 2m/M$ for electron elastic collisions with neutrals of mass M)
- ν = electron-neutral particle collision frequency.

Since $\tau_{EN} = 1/G\nu$ is the energy relaxation time for the electron gas, this condition may be written:

$$\tau_{EN} \gg T,$$

where $T = 2\pi/\omega$ is the period of the impressed field. Because the energy relaxation time for the electron gas is much greater than the period of the impressed electromagnetic field, the electron temperature will settle down to some average value dependent upon the mean square value of the electromagnetic field. In the steady state the electromagnetic propagation characteristics of a plasma perturbed by a relatively high-power electromagnetic field may be determined by specifying the complex dielectric constant, which is a function of the following parameters:

(1) An effective electron collision frequency

$$\nu_{EFF} = \frac{\sqrt{2}}{3\sqrt{\pi}} \left(\frac{m}{kT_e}\right)^{5/2} \int \nu(v)\, v^4 \exp\left(\frac{-mv^2}{2kT_e}\right) dv$$

$$\nu_{EFF} = \nu_{max}(T_e) + \frac{2}{3}\, T_e \frac{\partial \nu_{max}(T_e)}{\partial T_e} \quad , \tag{2}$$

where

$$\nu_{max}(T_e) = 4\pi \int \nu(v)\, f_{max}\, v^2 dv$$

$$= 4\pi \left(\frac{m}{2\pi kT_e}\right)^{3/2} \int \nu(v)\, v^2 \exp\left(\frac{-mv^2}{2kT_e}\right) dv \, .$$

$\nu(v)$ is the electron collision frequency as a function of electron velocity and T_e is the electron temperature.

(2) The interelectron collision frequency

$$\nu_{ee} = \frac{N_e \ln \Lambda}{11.4\, A^{1/2}\, T_e^{3/2}} \quad , \tag{3}$$

where $(\ell n \Lambda)$ is given as a function of N_e and T_e by Spitzer.[5] $A = 1/1823$ for electrons.

(3) The electron density N_e.

(4) The isotropic part of the electron velocity distribution function f_o (which is very nearly Maxwellian even in the presence of a strong electromagnetic field if the interelectron collision frequency is much greater than the collision frequency for energy transfer between the electron gas and the neutral-particle gas $\nu_{ee} \gg G\nu$).

It turns out that the condition $\nu_{ee} \gg G\nu$ is satisfied for the partially-ionized (0.01 to 0.1%) plasma behind strong shocks in air over a considerable range of pressures and temperatures. Peskoff[6] has tabulated the electron density, the inter-electron collision frequency, and the electron-neutral collision frequency for the stagnation region behind a normal shock with a free-stream velocity of 18,000 ft/sec at altitudes of 40, 60, 80, and 100 km. Peskoff's tabulations indicate that the inter-electron collision frequency (ν_{ee}) is of the order of the electron-neutral collision frequency (ν_{eN}); i.e., $\nu_{ee} \approx \nu_{eN}$. Since the fractional energy loss of an electron per collision (G) with the atoms and molecules found behind a high-temperature shock in air ranges between 10^{-3} to 10^{-2} for electron energies of 0.5 to 2 ev (corresponding to electron temperatures of 5000° to 20,000°K), the condition $\nu_{ee} \gg G\,\nu_{eN}$ is always satisfied behind a shock with gas temperatures at 5000°K and electron temperatures of 5000° to 20,000°K (see Massey and Burhop,[7] p. 279). It will be demonstrated in the next section (see Ginzburg and Gurevich,[8] p. 132) that when the condition $\nu_{ee} \gg G\nu$ is satisfied, the isotropic part of the electron velocity distribution function in a partially-ionized plasma is Maxwellian, even in the presence of a strong electromagnetic field. This fact permits the establishment of the electron temperature as a proper thermodynamic variable. If the additional assumption is made that the gas temperature is not changed by the perturbing electromagnetic field (a good approximation), then the steady-state electron density may be found as a function of the various parameters that determine the rate at which electrons appear or recombine. The rate coefficients for electron production or loss, such as the rate of ionization for electron impact on neutrals, dissociative recombination, and three-body recombination are known as an explicit function of electron temperature for the various constituents of high-temperature air.

Since the predominant electron production and electron loss mechanisms are believed to be known for a high-temperature shock in air, even when the electron temperature is greater than the neutral and heavy ion temperature, the computation of the change in the electron density of the plasma sheath produced by changes in the electron temperature will require specific knowledge of the electron temperature distribution in the slab of plasma. In the steady state the electron temperature at each point in the plasma slab depends upon the rate at which the electromagnetic field heats the electron gas and the rate at which the electron gas loses energy. The various electron energy loss mechanisms include:

(1) Elastic electron-neutral and electron-ion collisions.

(2) Ionizing collisions by electron impact.

(3) Collisions between electrons and molecules that involve the excitation of vibrational or rotational states.

(4) Recombination collisions (dissociative and three body).

(5) Heat transport due to gradients of the <u>difference</u> between the electron temperature and the neutral gas temperature. There are <u>two</u> heat transport coefficients (see Anderson and Goldstein,[9] p. 77). One transport coefficient depends upon heat conduction due to electron-neutral collisions and the other depends upon heat conduction due to electron-electron collisions (Spitzer and Harm[10]).

(6) Heat loss through electron diffusion which involves a term in the energy balance equation of the form $\frac{64k^2 T_e}{9\pi m \nu_{EFF}} \nabla N_e \cdot \nabla T_e$.

On the basis of equilibrium-flow calculations performed by Rotman and Meltz[11] for the electron density and temperature profile behind a high-temperature shock in air, it may be shown that the various heat loss mechanisms of the electron gas due to conduction or diffusion are negligible in comparison with losses due to elastic and inelastic electron-neutral and electron-ion collisions. The model assumed consists of a plane wave incident on a plasma slab, and owing to the infinite extent of the plane wave front, heat losses due to electrons flowing out of the region heated by the electromagnetic field will not be considered. These preliminary facts permit computation of the total reflection and transmission coefficient of the plasma sheath for relatively high-power densities (about 100 watts/cm^2) as a function of the amplitude of the incident plane wave, angle of incidence, and angle of polarization.

One of the first investigations of the nonlinear interaction of electromagnetic radiation with the plasma sheath surrounding hypersonic re-entry vehicles was conducted by Sisco and Fiskin.[15] In this study, the assumption was made that only the collision frequency of the hypersonically-produced plasma would change under perturbation by an electromagnetic wave as long as the field strengths involved were below the breakdown threshold. However the condition for breakdown may not be sharply defined in the case where ionization exists prior to the application of high-power electromagnetic radiation. It is possible that the electron density may change by a factor of two or three in a plasma layer surrounding an aperture antenna because of changes in the power level. Such alterations in the electron density will have as pronounced an effect on the microwave conductivity of the plasma as changes induced in the electron collision frequency. At power levels at X-band (10 kMc) of the order of 100 watts/cm^2, it is estimated that the electron temperature will change by about a factor of four (from 5000° to 20,000°K). This will produce about factor of two changes in electron density and collision frequency. If the frequency of a plane wave incident upon a plasma slab is about equal to the average plasma frequency in the slab ($\omega \approx \omega_p$), then there will be little reflection from the first air-plasma interface and a maximum of energy coupling between the electromagnetic wave and the lossy plasma. Alterations in the electron density and collision frequency of the order of a factor of two are capable of inducing 50 to 100 percent changes in reflection coefficient and absorption coefficient.

The method of computing the reflection and transmission coefficients of the nonlinear slab of plasma proceeds on the basis of a step-by-step numerical integration of Maxwell's equations. The approximation consists in replacing the inhomogeneous nonlinear plasma slab with a set of homogeneous linear slabs. If Maxwell's equations are written in the form of difference equations, and the step-by-step numerical integration is accomplished by using backward differences starting on the face of the slab from which the transmitted wave emerges, then the solution for the field distribution in the nonlinear, nonhomogeneous slab may be determined by assuming a value for the amplitude of the transmitted wave. In this scheme the fields are completely determined at the point Z-h, once the fields are known at the point Z and the dielectric constant is known at the point Z. 'h' is the increment chosen in obtaining solutions of the difference equations and corresponds to a layer over which the dielectric constant is assumed not to vary. The dielectric constant at the point Z in the plasma slab is found by calculating the effective collision frequency and the electron density at this point in the plasma. These parameters are determined once the electron temperature is known. The electron temperature is determined from an energy balance equation for the layer lying between the points Z and Z-h. The energy balance equation is simply a statement of the fact that, in the steady state, the rate at which the electromagnetic field supplies energy to the electron gas is equal to the rate at which the electron gas loses energy to the neutral and ion gas through elastic and inelastic collisions.

The step-by-step numerical integration of the electromagnetic field equations has been discussed by Penico,[12] Stickler,[13] and Richmond[14] for the case of a plane wave incident upon a dielectric slab with the dielectric constant a function of position in the direction perpendicular to the interfaces. Most of these analyses start with the wave equation for the field components. However an advantage is gained by starting from Maxwell's equations directly, particularly in the nonlinear problem. Specifically, Richmond[14] utilizes the wave equation for the field components E and H. In his analysis the solution of the difference equation is started by assuming a value for the amplitude of the transmitted field. Since the wave equation is of the second order, the field must be known at a second point to uniquely specify the solution. The field at the position of the first backward difference is usually expanded in a Taylor series with the size of the increment (h) as parameter. Determination of the field at the position of the first backward difference by a Taylor expansion is not necessary if Maxwell's equations (first-order equations) are used directly. It should be noted that the step-by-step numerical integration of the electromagnetic field equations by backward differences is a method that may be applied to the nonlinear plasma slab problem only when thermal and particle diffusion from layer to layer can be neglected. The effects of the boundaries are neglected insofar as it is assumed that the temperature and electron density gradients in the boundary layer and at the edge of the shock are so steep compared to a wavelength that the boundaries may be represented as plane interfaces.

2. THE CONDUCTIVITY OF A PARTIALLY IONIZED PLASMA IN THE PRESENCE OF A STRONG ELECTROMAGNETIC FIELD

If an electromagnetic field is impressed upon a partially-ionized gas, the electrons will gain energy from the electromagnetic field by suffering collisions with the neutral species and heavy ions in such a manner that their ordered oscillatory motion is changed to random thermal motion. For a partially-ionized plasma, such as high temperature air (5000°K), the electrons lose only a small relative fraction of their energy (G) per collision with neutrals and ions even for electron energies up to 2 ev($G \approx 10^{-3}$ at electron energies of 0.5 ev and $G \approx 10^{-2}$ at electron energies of 2 ev). Thus it is a good approximation to assume that the velocity distribution function for the neutral atoms and molecules and the ions remains Maxwellian and that their temperature remains constant even in the presence of relatively strong electromagnetic fields.

Initially, in the absence of an externally impressed electromagnetic field, the electrons are in thermal equilibrium with the neutrals and positive ions behind the high-temperature shock of a re-entry vehicle (this is true within about 100 mean free paths behind the shock front). Hence the electron velocity distribution function f is Maxwellian in the absence of an electromagnetic field and the electron temperature is equal to the gas temperature $T_e = T$. Consider a c.w. electromagnetic plane wave incident upon the plasma sheath with a frequency (ω) much greater than the electron-neutral collision frequency for energy transfer ($G\nu \approx 10^{+6}$ at $T_e = 5000$°K, $G\nu \approx 10^{+7}$ at $T_e = 20,000$°K for $T = 5000$°K, $\rho/\rho_o = 10^{-3}$ corresponding to a shock with free stream velocity = 18,000 ft/sec at 200,000 feet). In this case, the relaxation time for energy transfer between the electron gas and neutral gas (τ_{EN}) is much greater than the period of the impressed electromagnetic wave $\tau_{EN} >> T_{em}$, so that the average energy gained by the electron from the electromagnetic field will depend upon the mean square value of the field. The increase in the average energy of the electron, resulting from randomization of the electron's oscillatory motion through elastic collisions with neutrals, will induce changes in the various electron energy loss processes such as:

(1) Collisions which can excite electronic levels of atoms and/or molecules
(2) Collisions which can excite rotational and vibrational levels of molecules
(3) Ionizing collisions of atoms or molecules
(4) Dissociative recombination of ionic molecules and three-body recombination collisions with ionic atoms.

In addition to the elastic and various types of inelastic collisions of the electrons with heavy particles, the effect of electron-electron collisions must also be considered. Electron-electron collisions result in only a small relative fractional energy loss per collision

$$G_{ee} = \left(\frac{e^2 N_e^{1/3}}{k\,T_e}\right)^3 . \qquad (4)$$

This is a consequence of the fact that the total collision cross section for electron-electron collisions is obtained by integrating the Rutherford differential scattering cross over all angles down to a minimum scattering angle, corresponding to a maximum impact parameter equal to the Debye length (see Ginzburg,[8] p. 129).

$$\text{Debye length} = \ell_D = \left[\frac{kTkT_e}{4\pi N_e e^2 (kT + kT_e)}\right]^{1/2} . \qquad (5)$$

The fractional momentum loss for electron-electron collisions is of the same order as the fractional energy loss for electron-electron collisions. Spitzer[5] gives the following expression [Eq. (3)] for the interelectron collision frequency

$$\nu_{ee} = \frac{N_e \, \ell n \, \Lambda}{11.4\, A^{1/2} T_e^{3/2}} \quad \text{SEC}^{-1} \quad ,$$

where A = 1/1823 for electrons and $\ell n \, \Lambda$ is tabulated as a function of N_e and T_e.

For a fairly wide range of re-entry conditions, especially in and near the stagnation region of vehicles traveling at 18,000 ft/sec or greater at altitudes between 300,000 and 100,000 ft, the interelectron collision frequency (ν_{ee}) is several orders of magnitude greater than the collision frequency for energy transfer between the electron gas and the gas of heavy particles ($\nu_{ee} \gg G\nu$). When this condition prevails, any perturbation such as an electromagnetic wave applied to the partially-ionized plasma will result in the redistribution of energy among the electrons in a time $\tau_{ee} = 1/\nu_{ee}$ which is much faster than the time for energy transfer between the electron gas and the heavy particle gas ($1/G\nu$).

Such an effect will have a pronounced influence on the response of a plasma to a dynamic perturbation. The dynamic (time dependent) response of a re-entry plasma sheath under the influence of high-power electromagnetic radiation has been carefully analyzed by King.[16] King's investigation proceeds from the WKB solution of the wave equation for an electromagnetic wave incident upon a semi-infinite plasma. The validity of the WKB solution is restricted to cases where the changes in electron density (or dielectric constant K) are small compared to a wavelength $dK/dZ \ll K/\lambda$. The WKB method was developed in connection with obtaining solutions to the Schroedinger equation. In this class of problems the DeBroglie wavelength is almost always very small compared to distances over which the potential function varies. Also, with reference to ionospheric propagation, the criterion for the validity of the WKB approximation is met for frequencies in the megacycle range and higher. However the criterion may not always be met for electromagnetic propagation through the re-entry sheath, where the electron density may vary by ten orders of magnitude over a distance of about 1 cm with wavelengths in the centimeter range (Rotman and Meltz[11]). The WKB solution contains no information on the amplitude of the reflected wave, because reflection is neglected in the approximation of geometric optics. Ginzburg[17] has indicated a modification of the WKB method to include the effects of weak reflection.

The starting point in King's[16] analysis is the WKB solution for the electromagnetic field distribution in the re-entry plasma sheath. The WKB solution is written in the form

$$E = E_o \sqrt{k_o/k}\ \exp\left(\pm i \int_{x_o}^{x} k\, dx\right) . \tag{6}$$

where

$$k^2 = k_o^2 (\epsilon/\epsilon_o - i\, \sigma/\omega\epsilon_o)$$

$$k_o = \omega/c = 2\pi/\lambda_o$$

$$\epsilon/\epsilon_o = 1 - \frac{\omega_p^2}{\omega^2+\nu^2} ,$$

$$\sigma = \frac{N_e e^2 \nu}{m(\omega^2+\nu^2)} .$$

σ is the real part of the conductivity and the propagation direction in the direction of the electron density gradient (x-direction). This form for the field distribution is then substituted into the expression for energy deposition per unit time in the plasma (σE^2).

The fraction of the total energy deposited in the plasma which is responsible for producing increased ionization is given by $f\sigma E^2$ where $f = \sigma_i/\sigma$ and σ_i is the ionization cross section while σ is the total cross section. If this quantity is then divided by the energy required to form an ion pair in the plasma, ϵ, the result yields the local rate of growth of electrons due to the electromagnetic field:

$$\frac{\partial N_e}{\partial t} = \frac{f}{\epsilon} \sigma E^2 . \tag{7}$$

The electron density appears explicitly on the left-hand side and implicitly on the right-hand side of this equation. King[16] has obtained analytic expressions for the time dependence of density by making a suitable transformation of variables. The effects of diffusion and electron attachment may also be taken into account, although the solutions may no longer be expressed in terms of known functions. The implicit assumption of such an approach rests upon the relation.

$$\omega >> \frac{1}{K} \frac{\partial K}{\partial t} = \frac{1}{N_e} \frac{\partial N_e}{\partial t} ,$$

where K is the effective dielectric constant

$$K = K_r + jK_i = 1 - \frac{\omega_p^2}{\omega^2 + \nu^2} - \frac{j(\nu/\omega)\,\omega_p^2}{\omega^2+\nu^2}$$

$$\omega_p^2 = N_e \frac{e^2}{m\epsilon_o} .$$

For the case of a medium with a time-dependent dielectric constant, Maxwell's equations are:

$$\text{curl}\,\vec{E} = -\mu_o \frac{\partial \vec{H}}{\partial t}$$

$$\text{curl}\,\vec{H} = \epsilon_o \frac{\partial K\vec{E}}{\partial t} . \tag{8}$$

This leads, in general, to a wave equation of the form:

$$-\nabla^2\vec{E} - \vec{\nabla}\,[\nabla \vec{\ell n} K \cdot \vec{E}] = -\frac{1}{c^2} K \frac{\partial^2 \vec{E}}{\partial t^2} - \frac{2}{c^2} \frac{\partial K}{\partial t} \frac{\partial \vec{E}}{\partial t} - \frac{1}{c^2} \frac{\partial^2 K}{\partial t^2} \vec{E} .$$

The WKB approximation is a valid solution of the wave equation only if

$$\omega >> \frac{\partial \ell n N_e}{\partial t} ,$$

which permits the terms

$$\frac{2}{c^2}\frac{\partial K}{\partial t}\frac{\partial \vec{E}}{\partial t} \quad \text{and} \quad \frac{1}{c^2}\left(\frac{\partial^2 K}{\partial t^2}\right)\vec{E}$$

to be neglected and a solution to be obtained in the form $E \propto A(t)\, e^{j\omega t}$. The condition $\omega >> \partial \ell n N_e / \partial t$ implies that the characteristic time for electron density buildup is much longer than the period of the impressed field. This condition is almost always satisfied for most plasma-microwave interactions. A final point should be made that no provision for changes in the collision frequency have been included in the analysis, although such changes may generally be overshadowed by changes in the electron density.

Only the steady-state response of a re-entry plasma sheath to high-power electromagnetic radiation will be considered in this report. The question of the relative importance of the relaxation time for equilibration of energy among the electrons ($1/\nu_{ee}$) and the relaxation time for equilibration of energy between the electrons and neutrals and ion ($1/G\nu$) is important from the point of view of the dynamic response of the plasma to the perturbation of an electromagnetic wave. However, even in the steady state, electron-electron collisions can play a significant role in determining the form of the isotropic part of the electron velocity distribution function. The general form for the isotropic part of the electron distribution function in a partially-ionized plasma in the presence of a strong electromagnetic field is not always given by the Margenau[18] expression

$$f_o = c \exp\left[-\int_o^v \frac{mvdv}{kT + \dfrac{2e^2E^2}{3m\,G(\nu^2+\omega^2)}}\right], \tag{9}$$

where C = a normalizing factor
T = gas temperature
ν = electron-neutral collision frequency (a function of electron velocity).

The Margenau expression for the electron distribution function was derived under the assumption that electron-electron collisions are negligible ($\nu_{ee} << G\nu$). However, as pointed out by Cahn[19] and by Ginzburg and Gurevich,[8] when the inter-electron collision frequency is of the order of the collision frequency for energy transfer between electrons and neutrals ($\nu_{ee} \approx G\nu$), the form of the isotropic part of the electron velocity distribution function is given by a rather complex expression which reduces to the Margenau expression when $\nu_{ee} << G\nu$ and to the Maxwellian form when $\nu_{ee} >> G\nu$.

2.1 The Boltmann Equation for a Partially Ionized Gas

The propagation characteristics of electromagnetic waves in a finite layer of plasma may be described by a complex dielectric constant (or conductivity) which is a scalar quantity in the absence of a magnetic field. The conductivity (σ) is computed by taking the first velocity moment of the electron velocity distribution function:

$$\sigma = \frac{e}{E} \int f \vec{v} \, d^3v \quad . \tag{10}$$

Ginzburg[17] and Molmud[20] have shown that Eq. (10) for the complex conductivity reduces to the standard expression

$$\sigma = \sigma_r + j\,\sigma_i = \frac{N_e e^2 \nu}{m(\omega^2+\nu^2)} - j\,\frac{N_e e^2 \omega}{m(\omega^2+\nu^2)} \tag{11}$$

only if the collision frequency in Eq. (11) is replaced by an 'effective' collision frequency (ν_{EFF}) given by Eq. (2). The effective collision frequency, as given by Eq. (2), is valid only if $\omega^2 >> \nu^2$. For the low-frequency limit ($\omega^2 << \nu^2$), the effective collision frequency is defined differently. At intermediate frequencies, the right-hand side of Eq. (11) must be multiplied by a parameter that is a function of ω / ν_{EFF}. To utilize the exact Expression (10) for the conductivity, the form of the electron velocity distribution function must be found by solving the Boltzmann equation.

The full description of a multicomponent system such as a partially-ionized plasma involves a series of coupled Boltzmann equations for the distribution function of each component.

$$\frac{\partial f_e}{\partial t} + \vec{v} \cdot \vec{\nabla}_r f_e - \frac{e}{m_e}[\vec{E} + \vec{V} \times \vec{B}] \cdot \vec{\nabla}_v f_e = B_{ee} + \sum_j B_{e\text{-}ion_j} + \sum_k B_{en_k}$$

$$\frac{\partial (f_{ion})_\ell}{\partial t} + \vec{V} \cdot \vec{\nabla}_r (f_{ion})_\ell + \frac{Ze}{(m_{ion})_\ell}[\vec{E} + \vec{V} \times \vec{B}] \cdot \vec{\nabla}_v (f_{ion})_\ell = B_{e\,ion_\ell} + \sum_{k=1}^{n-1} B_{ion_\ell\,ion_k} + \sum_j B_{ion_\ell\,n_j}$$

$$\frac{\partial f_{nj}}{\partial t} + \vec{V} \cdot \vec{\nabla}_r f_{n_j} = \sum_k B_{n_j\,ion_k} + B_{en_j} + \sum_{k=1}^{n-1} B_{n_j n_\ell} \tag{12}$$

where $(f_{ion})_\ell$ is the distribution function for the ℓ^{th} ion constituent, f_{nj} is the distribution function for the j^{th} neutral constituent, the subscript e refers to electrons, and the subscript n refers to neutrals. B_{mn} represents the contribution to the collision integral for the distribution function of the m^{th} type of particles in collision with an n^{th} type. The Boltzmann equations for the electron, ion, and neutral components are coupled through the collision integral terms. The usual assumption made in investigations of the interaction of radio frequency or microwave energy with a partially-ionized gas is that only the electron distribution function changes, and the neutral and ion distribution functions are assumed to remain Maxwellian at a constant temperature. This assumption is valid when the collision frequency is relatively small compared to the plasma frequency, the plasma is less than 1% ionized and when the fractional energy loss of an electron per collision with a neutral is small ($G \leq 10^{-2}$ for electron energies below 2.5 ev for most gases).

The macroscopic fields that appear in Eq. (12) are self-consistent, so that the electrons do not interact directly with one another on a microscopic level (except for distances less than the Debye length, ℓ_D). Instead, the ensemble of electrons give rise to a macroscopic electromagnetic field which then can interact with the individual electrons. The macroscopic fields that appear on the left-hand side of the Boltzmann equation remain constant during a collision. This implies that, when considering electron-electron collisions, the sphere of interaction must be less than a Debye length (see Drummond,[21] p. 12).

The great majority of papers on the kinetic theory of electromagnetic wave interaction with a partially-ionized gas neglect ion-electron and electron-electron collisions (Holstein,[22] Margenau,[18, 23] Reder and Brown,[24] Allis and Brown[25]). Cahn[19] has discussed the effects of electron-electron and electron-neutral collisions on the distribution function. Ginzburg and Gurevich[8] have clearly indicated the conditions under which it is permissible to neglect electron-electron and electron-ion collisions in a partially-ionized gas. Elastic collisions between electrons and neutrals may be treated in the most straight-forward manner. The expression for the collision integral for electron-neutral collisions is obtained in terms of the neutral particle density, temperature, elastic collision cross section, and the zero-order electron energy distribution function. The derivation of these expressions usually involves the assumption that the electron suffers only a small change in energy on collision with a neutral (which is not necessarily true for excitation or ionizing collisions). Allis[26] has treated the case where account is taken of recoil of the heavy molecule under elastic collisions with electrons. Inelastic collisions between electrons and neutrals are accompanied by the excitation of rotational, vibrational, or electronic levels and also by ionization and recombination. In addition, charge exchange between ions and second-order impacts are possible, in which the energy of an excited state of the molecule is transferred to incoming electrons.

However, an exact calculation taking account of all these inelastic processes would be exceedingly complex. The cross sections for many of these processes are known exactly only in a few cases. In this report, the effects of charge exchange and second-order impacts (inelastic collisions of the second kind) are neglected.

The Boltzmann equation for the electron velocity distribution function for a partially-ionized plasma (the re-entry sheath) in the absence of a magnetic field may be written:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla}_r f + \frac{e}{m} \vec{E} \cdot \vec{\nabla}_v f = B_{ee} + \sum_i B_{ei} + \sum_j B_{en_j} \quad , \tag{13}$$

where E is the amplitude of the externally impressed electromagnetic wave and the forces exerted on the electrons by the magnetic component of the wave are neglected. Since diffusion is neglected in comparison with other electron loss mechanisms, the Boltzmann equation may be solved for the case of a homogeneous plasma within each section of the plane-layered medium. Hence, the Boltzmann equation may be written:

$$\frac{\partial f}{\partial t} + \frac{e}{m} \vec{E} \cdot \vec{\nabla}_v f = B_{ee} + \sum_i B_{ei} + \sum_j B_{en_j} \quad . \tag{14}$$

Ginzburg and Gurevich[8] have shown that when the electron energy loss parameter G is much less than one, a spherical harmonic expansion of the electron distribution function converges rapidly, so that the expansion may be terminated after the first two terms

$$f(\vec{r}, \vec{v}, t) = \sum_l f_l (\vec{r}, v, t) P_l \ (\cos \alpha) = f_0 + f_1 \cos \alpha \ , \tag{15}$$

where the electric field vector is taken to lie along the Z-axis and α is the angle between $\vec{E}$ and $\vec{v}$.

Using the spherical harmonic expansion [Eq. (15)], together with the expression for partial derivatives

$$\left(\frac{\partial}{\partial v_z}\right)_{v_x, v_y} = \cos \alpha \left(\frac{\partial}{\partial v}\right)_\alpha + \frac{\sin^2 \alpha}{v} \left(\frac{\partial}{\partial \cos \alpha}\right)_v \tag{16}$$

one obtains

$$\frac{e\vec{E}}{m} \cdot \vec{\nabla}_v \sum f_l P_l = \frac{eE}{m} \cos \alpha \frac{\partial f_l}{\partial v} P_l + \frac{eE}{m} \frac{f_l}{v} \sin^2 \alpha \cdot \frac{\partial P_l}{\partial (\cos \alpha)} =$$

$$= \frac{eE}{m}\left[\frac{1}{3v^2}\frac{d}{dv}v^2f_1 + \left(\frac{df_o}{dv} + \frac{2}{5}\frac{1}{v^3}\frac{d}{dv}v^3f_2\right)\cos\alpha + \ldots\ldots\right. \quad (17)$$

If Eq. (17) and Eq. (15) are substituted into Eq. (14), and if then the resulting equation is multiplied successively by P_o ($\cos\alpha$), P_1 ($\cos\alpha$), P_2 ($\cos\alpha$) etc., and integrated over $2\pi \sin\alpha\, d\alpha$, the resulting chain of coupled equations is obtained

$$\frac{\partial f_o}{\partial t} + \frac{eE}{3mv^2}\frac{\partial}{\partial v}\left(v^2 f_1\right) = B_o \quad (18a)$$

$$\frac{\partial f_1}{\partial t} + \frac{eE}{m}\left[\frac{\partial f_o}{\partial v} + \frac{2}{5v^3}\frac{\partial}{\partial v}\left(v^3 f_2\right)\right] = B_1 \quad (18b)$$

$$\frac{\partial f_2}{\partial t} + \frac{eE}{m}\left[\frac{2}{3}v\frac{\partial}{\partial v}\left(\frac{f_1}{v}\right) + \frac{3}{7v^4}\frac{\partial}{\partial v}\left(v^4 f_3\right)\right] = B_2 \quad (18c)$$

where

$$B_\ell = \frac{2\ell+1}{4\pi}\int\left[B_{ee} + \sum_i B_{ei} + \sum_j B_{en_j}\right]\cdot P_\ell\ (\cos\alpha)\, d\Omega$$

and

$$d\Omega = 2\pi \sin\alpha\, d\alpha \quad .$$

Since the electron energies will be considered only up to 2 ev, for which $G \approx 10^{-2}$ for each of the constituents of high-temperature air, only the equations for f_o and f_1 need be considered.*

2.1.1 COLLISION INTEGRAL FOR ELASTIC COLLISIONS BETWEEN ELECTRONS AND NEUTRALS OR IONS

The collision integral may be evaluated by considering the rate of change of the number of particles in a velocity volume element d^3v. This rate of change may be found by taking the difference between the number of particles scattering into that volume element and the number scattered out:

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* Actually, $G = 10^{-1}$ at 2 ev for NO, but the NO concentration in a shock at 5000°K is less than or equal to 1/10 the concentration of N_2 and O_2.

$$B d^3v = \int F(\vec{V}')\, f(\vec{v}')\, c\, \sigma(\Theta, c)\, d^2\Omega\, d^3V' d^3v'$$

$$- \int F(\vec{V})\, f(\vec{v})\, c\, \sigma(\Theta, c)\, d^2\Omega\, d^3V d^3v \ , \tag{19}$$

$F(V)$ = distribution function for scattering particles
$f(v)$ = distribution function for scattered particles
c = relative speed of particles
$= |\vec{v} - \vec{V}| = |\vec{v}' - \vec{V}'|$
σ = differential scattering cross section
$d^2\Omega$ = $2\pi \sin\Theta\, d\Theta$
Θ = scattering angle = angle between $\vec{v} - \vec{V}$ and $\vec{v}' - \vec{V}'$.

The primes refer to the quantities before a collision and the unprimed quantities refer to after a collision. By Liouville's theorem

$$d^3v' d^3V' = d^3v d^3V \ .$$

so that the collision integral may be written:

$$B = \int [F(\vec{V}')\, f(\vec{v}') - F(\vec{V})\, f(\vec{v})]\ c\, \sigma d^2\Omega d^3V. \tag{20}$$

Allis[26] has indicated how the spherical harmonic components of the collision integral may be evaluated. If the spherical harmonic expansion for $f(v)$ [Eq. (15)] is substituted into Eq. (20), and use is made of the addition theorem for spherical harmonics,

$$f(\vec{v}') = \sum_l f_l(v') \left[P_l(\alpha) P_l(\Theta) + 2 \sum_m \frac{(l-m)!}{(l+m)!}\, P_l^m(\alpha) \cdot P_l^m(\Theta) \cos \vec{m}(\alpha - \Psi) \right], \tag{21}$$

where Ψ = azimuthal angle, then the Boltzmann collision integral may be written

$$B = \sum_l B_l P_l(\cos\alpha)$$

where

$$B_l = \int \left(\frac{|\vec{v}' - \vec{V}'|}{|\vec{v} - \vec{V}'|} \right)^3 F(\vec{V}')\, f_l(v')\, P_l(\Theta) \cdot c\sigma d^2\Omega d^3V' - \int F(\vec{V})\, f_l(v) c\sigma d^2\Omega d^3V. \tag{22}$$

The zero-order term in the spherical harmonic expansion of the electron-heavy particle collision integral representing elastic collisions may be written

$$B_o^{elas.} = \frac{2m}{M+m} \frac{1}{v} \frac{d}{dv^2} \left[v^3 \nu \left(f_o + \frac{2kT}{m} \frac{df_o}{dv^2} \right) \right]$$

$$= \frac{1}{2v^2} \frac{\partial}{\partial v} \left[v^2 G_{EL} \nu \left(\frac{kT}{m} \frac{\partial f_o}{\partial v} + v f_o \right) \right] \tag{23}$$

where m = electron mass
M = heavy particle mass
T = temperature of heavy particle gas
G = 2m/M+m = relative fraction of the energy loss of an electron per collision with heavy particle

$\nu = N v \int \sigma(v, \theta)(1-\cos\theta) 2\pi \sin\theta d\theta.$

Allis[26] has derived Eq. (23) on the basis of the assumption that the electron loses only a small fraction of its energy in colliding with a heavy neutral particle ($G \ll 1$). Also, it is assumed that the molecules recoil under electronic impact and they possess a Maxwellian distribution in velocity.

The first spherical harmonic component of the collision integral representing elastic electron-neutral collisions may be immediately derived by using the fact that the magnitude of the electron's velocity changes only slightly during an elastic collision with a heavy particle ($v' = v$ and $V' = V$), so that

$$B_1^{elas.} = \int \sigma(\theta, c)\, c\{f_1(v')\, F(V')\, P_1(\cos\theta) - f_1(v)\, F(V)\}\, d^2\Omega d^3V$$

$$= -f_1(v) \int d^3V d^2\Omega \sigma v F(V)(1-\cos\theta)$$

$$= -\nu(v)\, f_1(v) \tag{24}$$

where

$$\nu \equiv Nv \int \sigma(v, \Theta)(1-\cos\Theta)\, d^2\Omega$$

where N = neutral particle density and ν is the collision frequency for momentum transfer.

2.1.2 COLLISION INTEGRAL FOR INELASTIC COLLISION BETWEEN ELECTRONS AND NEUTRALS

Inelastic collisions between electrons and neutral particles result in the excitation of rotational, vibrational, and/or optical levels of molecules. For a partially-ionized plasma consisting of diatomic gases such as oxygen, nitrogen, and NO, the dominant electron energy loss mechanisms for electrons of about 1 ev or lower energy are excitation of rotational and vibrational levels. The excitation of rotational levels results in an energy loss of 10^{-2} to 10^{-4} ev; whereas the excitation of vibrational levels results in electron energy losses of about 0.1 to 0.5 ev. If the electron energy distribution function is Maxwellian, then ionizing collisions can become important when the average electron energy is 1/10 or greater then the ionization potential of any atomic or molecular constituent (NO has the lowest ionization potential, 9.25 ev, of any constituent in high-temperature air). This is due to the long tail at high energies of the Maxwellian distribution function.

In the present report, electromagnetic fields incident on the re-entry sheath will be considered with intensities sufficient to raise the electron temperature from 5,000°K (0.5 ev) up to 20,000°K (2 ev). Measurements of the mean energy of electrons in swarm tube experiments together with Luxembourg (cross modulation) type experiments make it possible to deduce the energy loss for electrons in this energy range. Such experiments have been performed by Healey and Reed,[27] Harries,[28] Haas,[29] and Huxley.[30] Table 1 (from Massey and Burhop[7]) gives the fractional energy loss of electrons λ for the various constituents of air for electron energies between 0 and 6.0 ev.

Since, for an average electron, only a small part of the energy is lost below and up to excitation of rotational, vibrational, and optical levels, the integral for such inelastic collisions in a molecular plasma may be represented in the form (Ginzburg and Gurevich[8]):

$$B_o^{\text{inelas.}} = \frac{1}{2v^2} \frac{\partial}{\partial v} \left\{ G_{EFF} \nu^{\text{inel}} v^2 \left[\left(\frac{kT}{m} \frac{\partial f_o}{\partial v} + v f_o \right) \right] \right\} \tag{25}$$

where $G_{EFF}(V)$ describes the total relative fractional energy loss of an electron per collision due to excitation of rotational, vibrational and optical levels.

$$\nu^{\text{inel}} = N v \int \sigma_{\text{inel}} (v, \theta) (1 - \cos \theta)\, d^2\Omega$$

and σ_{inel} = cross section for such inelastic collisions. It may be noted that the relative fractional energy loss parameter (G) is known from experiment only as a function of electron temperature (T_e), and not electron velocity.

*TABLE 1. Average fractional energy loss λ due to collisions with gas molecules of electrons of different mean energy $\bar{\epsilon}$

Gas	Air		H_2	N_2	O_2	CO	NO	HCl	N_2O	CO_2	NH_2
$\bar{\epsilon}$(ev)	0.372		5.45	0.387	0.341	0.387	0.363	0.298	0.247	0.247	0.641
	(i)	(ii)				$\lambda \times 10^4$					
0.1	..	...	25	...	...	45	...	250	600	360	285
0.2	13	26	29	7	65	70	110	600	1,500	580	420
0.4	13	26	31	6	55	89	390	450	1,300	650	370
0.6	13	26	36	6	33	70	450	350	1,100	630	330
0.8	13	26	42	6.5	24	54	380	290	1,000	600	305
1.0	14	28	47	8.8	20	62	320	...	900	575	280
1.2	15	30	51	12.8	15	75	275	...	880	560	255
1.4	16	43	56	22.5	24	110	275	...	860	550	240
1.6	17	83	62	48	32	150	280	...	800	540	220
1.8	19	150	71	90	70	195	320	...	740	535	215
2.0	20	220	80	162	105	250	...	...	690	535	230
2.5	28	335	100	294	240	360	...	...	600	515	...
3.0	40	415	130	389	355	470	...	...	540	...	...
4.0	62	580	255	500	...	...	...	...	...	...	...
5.0	86	725	440	580	...	...	...	...	...	...	...
6.0	..	870	...	...	...	...	...	...	...	...	...

*from Massey and Burhop[7]

Here, $\lambda = G(1 - 4\epsilon_o/3\bar{\epsilon})$

ϵ_o = mean energy of gas molecules

$\epsilon_o = 1.3 \times 10^{-3}$ ev

$\bar{\epsilon}$ = mean energy of impacting electron

However this amount of information is sufficient for the present investigation, since the Boltzmann equation [Eq. (18)] will be integrated over the zero-order distribution function, which is Maxwellian. If cases are to be considered where the interelectron collision frequency (ν_{ee}) is not sufficiently high compared with the collision frequency for energy transfer ($G\nu$) to maintain a Maxwellian form for the isotropic part of the distribution function, then G must be known as a function of electron velocity v. This functional dependence may be obtained by solving the integral equation for G(v):

$$\int_0^\infty G(v)\,\nu(v)\,v^4 \exp\left[-\frac{mv^2}{2kT_e}\right] dv = \frac{3\sqrt{\pi}}{\sqrt{2}}\left(\frac{kT_e}{m}\right)^{5/2} G(T_e)\nu_{EFF}(T_e) \quad (26)$$

where the right-hand side of Eq. (26) is known.

The first spherical harmonic component of the inelastic collision integral representing electron impact collisions which excite rotational, vibrational, and optical levels in the molecular plasma may be written in the form:

$$B_1^{inelas} = -\nu^{inelas} f_1 \quad (27)$$

where

$$\nu^{inelas} = N\,v \int \sigma_{inel}(v,\theta)\,(1 - \cos\theta)\,d^2\Omega \,,$$

and $\sigma_{inel}(v,\theta)$ is the differential-scattering cross section for inelastic scattering.

2.1.3 COLLISION INTEGRAL FOR ELASTIC SCATTERING BETWEEN ELECTRONS AND POSITIVE IONS

The primary assumptions made in the derivation of Eqs. (23) and (24) for elastic collisions between electrons and heavy neutral particles were: (a) the fractional energy loss of the electron per collision is small, (b) the electron mass is very small compared with the neutral mass $m \ll M$, and (c) the temperature of the gas consisting of the heavy particles remains constant regardless of the average electron energy. Hence, Eqs. (23) and (24) for the zero- and first-order spherical harmonic components of the collision integral may also be used to describe elastic collisions between ions and electrons when the appropriate expression for the ion-electron collision frequency is used. The equation

$$\nu_{ion} = N_{ion}\,v \int \sigma(v,\theta)\,(1 - \cos\theta)\,d^2\Omega$$

governs the functional dependence of the electron-ion collision frequency on electron velocity, where $\sigma(v, \theta)$ is given by the Rutherford formula:

$$\sigma = \left(\frac{e^2}{2mv^2}\right)^2 \frac{1}{\sin^4 \theta/2}$$

where θ = scattering angle and
N_i = particle density of ions = N_e.

Thus,

$$\nu_{ion} = 2\pi N_e v \left(\frac{e^2}{2mv^2}\right)^2 \int_{\theta_{min}}^{\pi} \frac{(1 - \cos\theta)}{\sin^4 \theta/2} \sin\theta d\theta$$

$$= 2\pi N_e \frac{e^4}{m^2 v^3} \ln\left(1 + \cot^2 \theta_{min/2}\right) \tag{28}$$

where θ_{min} is the minimum angle of scattering corresponding to the maximum impact parameter. Since, as shown by Drummond,[21] collisions between charged particles which occur at distances greater than the Debye length are described by macroscopic fields which appear on the left-hand side of the Boltzmann equation, the maximum impact parameter will be taken equal to the Debye length [ℓ_D, Eq. (5)] :

$$\ell_D \tan \theta_{min/2} = \frac{e^2}{mv^2} \tag{29}$$

or

$$\theta_{min} = 2 \tan^{-1} \frac{e^2}{mv^2 \ell_D} \approx \frac{2e^2}{mv^2 \ell_D} .$$

Substituting Expression (29) for θ_{min} into Eq. (28) yields:

$$\nu_{ion}(v) = 2\pi N_e \frac{e^4}{m^2 v^3} \ln\left(1 + \frac{\ell_D^2 m^2 v^4}{e^4}\right) . \tag{30}$$

Bachynski et al.[31] have derived the expression for the contribution to the conductivity of a plasma by electron-ion collisions. Their computation proceeds from the formula

$$\sigma_{ion} = -\frac{4\pi e^2 N_e}{3m} \frac{\int f_1 v^3 dv}{\int\int g_z d^2\Omega v^2 dv} \quad . \tag{31}$$

where the vector $\vec{g}$ represents the flow velocity in velocity space (the collision integral is related to $\vec{g}$ through the relation $B = -\vec{\nabla}_v \cdot \vec{g}$). The vector $\vec{g}$ is evaluated by considering the diffusion in velocity space and computing the average of the velocity change of a test particle after an encounter. The computation depends upon the form of the electron distribution function. Bachynski et al.[31] have presented a treatment somewhat similar to that of Spitzer and Harm,[10] who have included the effects of electron-electron collisions on the conductivity. Both treatments are based upon the calculation of a transport coefficient (conductivity) in the presence of a DC electric field. The AC conductivity may be related to the DC conductivity by the approximate formula:

$$\sigma_{AC} \approx \frac{N_e e^2}{m\left(\frac{N_e e^2}{m\,\sigma_{DC}} + j\omega\right)} \quad . \tag{32}$$

It may be noted that Eq. (32) is valid only if $\nu_{ion} \approx (\nu_{ion})_{EFF}$, where $(\nu_{ion})_{EFF}$ may be found by substituting ν_{ion} [Eq. (30)] into Eq. (2):

$$(\nu_{ion})_{EFF} = \frac{2}{3}\sqrt{\frac{8\pi}{m}} \frac{e^4 N_e}{(kT_e)^{3/2}} \ln\left(\frac{kT_e \ell_D}{e^2}\right) \tag{33}$$

which is valid when $\omega^2 \gg \nu^2_{EFF}$.

Bachynski et al.[31] have expressed the total conductivity of a partially-ionized gas (σ_{total}) as the geometric mean of two conductivities – one due to electron-ion collisions (σ_{ion}) and another due to electron-neutral collisions (σ_n):

$$\frac{1}{\sigma_{total}} = \frac{1}{\sigma_{ion}} + \frac{1}{\sigma_n} \quad . \tag{34}$$

This relation is valid, of course, only for the DC case, and is based upon the assumption that the total collision frequency for electrons (ν_{total}) is given by

$$\nu_{total} = \nu_{ion} + \nu_{neutrals} \quad , \tag{35}$$

where $\nu_{neutrals}$ is the electron-neutral collision frequency.

As noted by Shkarofsky et al.,[32] in a later paper, a more satisfactory procedure than the relationship expressed by Eq. (34) exists for the computation of the total conductivity of a partially-ionized plasma taking into account electron-ion and electron-neutral collisions. If the expression for ν_{total} in Eq. (35) is used in the formula for the first spherical harmonic component of the collision integral $B_1 = -\nu_{total} f_1$, then this expression for B_1 may be substituted into Eq. (18b) for the first spherical harmonic component of the distribution function f_1. Equation (18b) is then solved for f_1 in terms of f_0, v, and ν_{total}. This solution is then substituted into Eq. (10), so that the conductivity is expressed as a function of f_0, the frequency of the impressed field ω, and ν_{total}. The electron-electron collisions produce two effects on the conductivity. First, if $\nu_{ee} >> G\nu$, than f_0 is Maxwellian even in the presence of strong electromagnetic fields. Second, there is a first spherical harmonic component of the electron-electron collision integral which introduces a correction to the total collision frequency appearing in Eq. (10) for the conductivity. This correction appears in the form (see Ginzburg,[17] p. 83):

$$\nu_{total} \rightarrow \nu_{total} (1 + \nu_{ee}/\omega) \quad , \tag{36}$$

where ν_{ee} is given by Eq. (3).

Shkarofsky et al.,[32] have computed the electron collision frequency of high-temperature air as a function of gas pressure and electron velocity by adding the electron-neutral collision frequencies for each constituent and then adding the ion-electron collision frequency [Eq. (30)] to obtain the total collision frequency:

$$\nu_{total} = \nu_{ion} + \sum_j N_j \sigma_j(v)v \ . \tag{37}$$

If $\omega \lesssim \nu_{ee}$, it should be noted that the correction term for interelectron collisions [Eq. (36)] is appreciable, and should be added before substitution into the expression for the conductivity, Eq. (10).

2.1.4 COLLISION INTEGRAL FOR ELECTRON-ELECTRON COLLISIONS

Electron-electron collisions are characterized by the long-range Coulomb forces which result in weak scattering. One of the primary differences between electron-neutral and electron-electron collisions consists in the ratio between fractional energy to fractional momentum lost per collision. For electron-neutral collisions, this ratio is G; while for electron-electron collisions, this ratio is one. For electron-electron collisions, both the energy and momentum are changed only slightly per collision, where the fractional energy loss of an electron per collision (G_{ee}) is given by Eq. (4).

An important consequence of the small change in energy and momentum of an electron in electron-electron encounters is the possibility of representing the collision integral as the divergence of a particle flux in velocity space (the Fokker-Planck expression):

$$B = -\vec{\nabla}_v \cdot \vec{g}$$

where the particle flux density is given by:

$$\vec{g} = \frac{1}{2}\iint c\,\Delta\vec{v}\,\sigma(c,\theta)\,\{f(\vec{v}\,')\,f(\vec{V}\,') - f(\vec{v})\,f(\vec{V})\}\,d^2\Omega\, d^3V \tag{38}$$

$$c = |\vec{v} - \vec{V}|$$

$$\Delta\vec{v} = \vec{v}\,' - \vec{v} \quad .$$

Since the velocity of an electron changes only slightly during an electron-electron encounter, the difference in the distribution functions before and after the collision may be written:

$$f(\vec{v}\,')\,f(\vec{V}\,') - f(\vec{v})\,f(\vec{V}) = [\Delta\vec{V} \cdot \overrightarrow{\nabla_V F}(\vec{V})]\;f(\vec{v}\,') - [\Delta\vec{v} \cdot \vec{\nabla}_v\, f(\vec{v})]\,f(\vec{V}) \;, \tag{39}$$

where $\Delta\vec{v} = \vec{v}\,' - \vec{v}$ and $\Delta\vec{V} = \vec{V}\,' - \vec{V}$. If Eq. (39) is substituted into Eq. (38) and use is made of the fact that $\sigma(c,\theta)$ has a maximum at $\theta = 0°$, then the particle flux density may be written:

$$\vec{g} = \frac{1}{2N_e}\int d^3V\,\nu_{ee}(c)\{\vec{c}\,[f(\vec{v})\,\vec{c}\cdot\overrightarrow{\nabla_V f}(\vec{V}) - f(\vec{V})\vec{c}\cdot\overrightarrow{\nabla_v f}(\vec{v})]$$

$$+ c^2\,[f(\vec{v})\,\overrightarrow{\nabla_V f}(\vec{V}) - f(\vec{V})\,\nabla_v \vec{f}(\vec{v})]\} \tag{40}$$

where $\vec{c} = \vec{v} - \vec{V}$

and

$$\nu_{ee}(c) = 2\pi N_e \frac{e^4}{m^2c^3} \ \ell n \left(1 + \frac{\ell_D^2 m^2 c^4}{e^4}\right) .$$

Ginzburg and Gurevich[8] have shown how Eq. (40) for the particle flux density may be used to derive the zero-order spherical harmonic component of the electron-electron collision integral:

$$B_o^{ee} = -\frac{1}{v^2} \frac{\partial}{\partial v} (v^2 g)$$

$$= \frac{1}{v^2} \frac{\partial}{\partial v}\left\{v^2 \left[A_1(f_o)\, v f_o + A_2(f_o) \frac{\partial f_o}{\partial v}\right]\right\} \qquad (41)$$

where terms of order f_1^2 have been neglected compared with terms f_o^2. Here,

$$A_1(v) = \frac{4\pi \nu_{ee}(v)}{N_e} \int_o^v V^2 f_o(V)\, dV$$

and

$$A_2(v) = \frac{4\pi \nu_{ee}(v)}{3N_e}\left[\int_o^v V f_o(V) dV + v^3 \int_v^\infty V f_o(V)\, dV\right] .$$

The collision integral B_o^{ee} identically equals zero when f_o is Maxwellian.

The first-order spherical harmonic component of the electron-electron collision integral may be obtained directly from Eq. (24):

$$\overrightarrow{B_1^{ee}} = \frac{3}{4\pi} \int \sigma(c, \theta)\, c \frac{\vec{v}}{v} \left[\frac{\vec{V}' \cdot \vec{f}_1(V')}{V'} f_o(v') + \frac{\vec{v}' \cdot \vec{f}_1(v')}{v'} f_o(V)\right.$$

$$\left. - \frac{\vec{v} \cdot \vec{f}_1(v)}{v} f_o(V) - \frac{\vec{V} \cdot \vec{f}_1(V)}{V} f_o(v)\right] d^2\Omega d^2K d^3V \qquad (42)$$

where θ is the angle between the relative velocities before and after the collision:

$$\vec{c}' = \vec{V}' - \vec{v}'$$

$$\vec{c} = \vec{V} - \vec{v}$$

$$d^2\Omega = 2\pi \sin \theta d\theta$$

$$d^2k = 2\pi \sin \beta d\beta .$$

β is the angle between $\vec{v}$ and $\vec{V}$ (in the laboratory frame). In the center-of-mass reference frame, the relative velocities of the two particles (before and after the collision) are equal in magnitude and opposite in sense: each particle in the center of mass having a velocity equal to $\frac{1}{2}/\vec{c}/$. As a result of conservation of momentum and energy $/\vec{c}'/ = /\vec{c}/$ and $\vec{V}' + \vec{v}' = \vec{V} + \vec{v}$. The velocity of the center of mass is equal to $\frac{1}{2}(\vec{V} + \vec{v})$. In the laboratory reference frame, the kinetics of the electron-electron collision may be represented by the three vectors: $\vec{c}' = \vec{V}' - \vec{v}'$, which is the relative velocity of the two electrons before the encounter, $\vec{c} = \vec{V} - \vec{v}$, which is their relative velocity after the encounter (equal in magnitude to their relative velocity before the encounter, but rotated by the angle θ), and the vector

$$\vec{d} = \vec{V}' + \vec{v}' = \vec{V} + \vec{v},$$

which is the sum of their velocities, and remains fixed in magnitude and direction before and after the encounter. $\sigma(c, \theta)$ is given by the Rutherford formula. Ginzburg[33] has evaluated the vector integral represented by Eq. (42), and demonstrated that the result may be written:

$$\vec{B}_1^{ee} = -\frac{\nu_{ee}}{\omega} \quad \nu \vec{f}_1(v) \tag{43}$$

where $\nu = \nu_{en} + \nu_{ion}$. This result is noteworthy, for it leads to a correction term for the total collision frequency: the sum of electron-neutral and electron-ion collision frequencies must be multiplied by the factor $(1 + \nu_{ee}/\omega)$ when Eq. (10) is used in the calculation of the conductivity.

2.1.5 COLLISION INTEGRALS FOR IONIZING AND RECOMBINATION COLLISIONS

For most gas discharge phenomena, such as arc and glow discharges, the primary electron production mechanism is ionization of neutrals by electron-impact. Bond[34] has found that electron-impact of neutrals is the chief ionization process behind shock waves in noble gases. On the basis of ionization cross sections of N_2, O_2, and NO deduced by Massey and Burhop[7] from data taken by Tate and Smith,[35] Lin and Teare[4] have computed the specific ionization rate due to electron impact on the constituents of high-temperature air as a function of distance behind a normal one-dimensional shock. By examining the relative importance of various electron production mechanisms behind strong shocks in air, Lin and Teare[4] have demonstrated that electron-impact is a relatively inefficient process compared with atom-atom and atom-molecule collisions. However, if in the presence of an electromagnetic field the electron-distribution function remains Maxwellian and the electron temperature becomes equal to or greater than 10,000°K (1 ev), then ionization by electron-impact must be considered. The dominant electron loss processes are dissociative recombination (with positive molecular ions) and three-body recombination (with positive atomic ions).

Margenau[23] has considered the collision integral representation for ionization and recombination collisions. To obtain an explicit expression for the ionization collision integral, it is necessary to make an assumption regarding the division of energy between the two electrons after the ionizing impact. Margenau[23] assumes that the energy is divided equally (note that Peskoff[6] takes into account the case where one electron after the ionizing collision comes off with zero energy):

$$(v')^2 = 2v^2 + u_i$$

$$v'dv' = 2v\,dv$$

where v' equals the velocity of the incident electron, $u_i = E_i/m$ and E_i is the ionization energy. Margenau expresses the collision integral for ionizing collisions by electron impact on a neutral constituent with a particle density (N_j) in the following form:

$$\int_o^v B_o^{\text{ioniz}}\, 4\pi v^2 dv = 4\pi N_j \left[2\int_v^{v'} f_o\, \sigma_{\text{ion}_j}\, v^3 dv + \int_o^v f_o\, \sigma_{\text{ion}_j}\, v^3 dv \right] . \qquad (44)$$

Here, σ_{ion_j} is the ionization cross section for the j^{th} neutral constituent. The first integral in Eq. (44) represents the production of two electrons with velocities less than v due to an impacting electron with velocity greater than v. The second integral represents the appearance of one electron due to ionization with an impacting electron having a velocity less than v. When Eq. (44) is integrated over all velocities, the total rate of electron production due to electron impact on the j^{th} neutral constituent is given by:

$$\left(\frac{dN_e}{dt}\right)_j = \int_o^\infty B_o^{\text{ioniz}}\, 4\pi v^2 dv = 4\pi N_j \int_o^\infty f_o\, \sigma_{\text{ion}_j}\, v^3 dv \quad . \qquad (45)$$

Lin and Teare[4] have written the expression for the rate of electron production in terms of the electron energy $\epsilon = \frac{1}{2}mv^2$:

$$\left(\frac{dN_e}{dt}\right)_j = N_e \left(\frac{8}{\pi m}\right)^{1/2} \left(\frac{1}{kT_e}\right)^{3/2} N_j \int_{W_{ij}}^{\infty} \exp\left(\frac{-\epsilon}{kT_e}\right) \sigma_{ion_j}(\epsilon)\epsilon d\epsilon \tag{46}$$

where W_{ij} is the threshold energy for the ionization cross section of the j^{th} constituent, and f_o is assumed to be Maxwellian:

$$f_o = N_e \left(\frac{m}{2\pi kT_e}\right)^{3/2} \exp\left(\frac{-mv^2}{2kT_e}\right) \tag{47}$$

Since NO has the lowest ionization potential of any constituent (equal to 9.25 ev) and the maximum electron temperature to be considered will be 20,000°K (2 ev), kT_e is always much less than the threshold energy for ionization (W_{ij}) so that an asymptotic expansion of Eq. (46) may be made:

$$\left(\frac{dN_e}{dt}\right)_j = N_e \left(\frac{8kT_e}{\pi m}\right)^{1/2} N_j \exp\left(\frac{-W_{ij}}{kT_e}\right) W_{ij}\ \sigma'_{ion_j}(W_{ij}) \tag{48}$$

where $\sigma'_{ion_j}(W_{ij})$ is the slope of the ionization cross-section curve vs energy at the threshold energy. By examining the curves of ionization cross section vs electron energy given by Massey and Burhop[7] for N_2 and O_2, Lin and Teare[4] conclude that, for these molecules,

$$\sigma'_{ion_j}(W_{ij})\, W_{ij} \approx \pi a_o^2 = 0.87 \times 10^{-16}\ \text{cm}^2$$

where a_o is the Bohr radius.

Since the ionization cross sections vs electron energy curves for NO, N_2, O_2, and the noble gases all exhibit the same shape (except for a displacement in the threshold energy), Lin and Teare take

$$\sigma'_{ion_j}(W_{ij})\, W_{ij} = 0.87 \times 10^{-16}\ \text{cm}^2$$

for the constituents N and O also.

The total rate at which electrons disappear due to recombination with positive ions of particle density N_i is given by an expression similar to Eq. (45):

$$-\left(\frac{dN_e}{dt}\right)_i = \int_0^\infty B_0^{recomb}\, 4\pi v^2 dv$$

$$= 4\pi N_i \int_0^\infty f_0 \sigma_{R_i}(v)\, v^3 dv$$

$$= 4\pi N_e N_i \left(\frac{m}{2\pi k T_e}\right)^{3/2} \int_0^\infty \sigma_{R_i}(v) \exp\left(\frac{-mv^2}{2kT_e}\right) v^3 dv \tag{49}$$

where N_i = particle concentration of i^{th} constituent of positive ions
and σ_{R_i} = recombination cross section for i^{th} ion constituent.
The isotropic part of the electron distribution function is assumed to be Maxwellian, so that the rate of disappearance of electrons due to recombination with the i^{th} constituent of positive ions is expressed as a function of electron density (N_e), positive ion density (N_i) and electron temperature (T_e). Now, the reaction rate for a two-body interaction such as dissociative recombination ($e + XY^+ \rightarrow X + Y$) is defined as:

$$k = -\frac{dN_e}{dt}\left[\frac{1}{(N_i)(N_e)}\right] . \tag{50}$$

By examining Eq. (49) and comparing it to Eq. (50), it may be noted that the reaction rate for dissociative recombination is a function only of the electron temperature. This important result is a consequence of the fact that the electron velocity is much greater than the velocity of the heavy ion. The most general expression for a reaction rate, which includes the case where the electron gas is not necessarily in thermal equilibrium with the ion gas, may be written in the form:

$$\frac{dN_e}{dt} = k\, N_e N_i = \iint f(V_e) f(V_i) \sigma(V_r) V_r d^3V_e d^3V_i \quad , \tag{51}$$

where V_e = electron velocity
V_i = ion velocity
V_r = ion-electron relative velocity.

Because the electron mass is always much less than the ion mass, $V_r = V_e$. Hence,

$$k = \int f(V_e)\, \sigma(V_e)\, V_e d^3V_e \quad . \tag{52}$$

Lin and Teare[4] claim that the predominant electron production mechanism behind strong shocks in air is atom-atom impact. Several of the most important reactions include:

$$
\begin{aligned}
&\text{(a)}\quad N + O + 2.8 \text{ ev} \rightarrow NO^{+} + e \\
&\text{(b)}\quad N + N + 5.8 \text{ ev} \rightarrow N_2^{+} + e \\
&\text{(c)}\quad O + O + 6.9 \text{ ev} \rightarrow O_2^{+} + e \ .
\end{aligned}
\tag{53}
$$

It may seem surprising that reactions (a), (b), and (c) are more important for electron production in air shocks than ionization by electron impact, since electron impact was found to be the predominant mechanism (several mean free paths behind the shock front where an appreciable electron concentration has built up) in Argon shocks (Bond[34]). The reactions [Eq. (53)] are efficient electron producers, even though the average kinetic energy of the atoms is considerably less than the energy of the endothermic reactions, because the presence of crossing of the potential energy curves of the colliding atomic system lead to a large cross section. However, direct electron impact on neutral constituents will start to compete with and even predominant over neutral-neutral impact, as an electron production mechanism when a high-intensity electromagnetic wave is impressed upon the ionized flow field. From an analysis of potential energy curve crossing, in conjunction with a somewhat heuristic curve-fitting procedure, Lin and Teare[4] arrive at an estimate for the forward reaction rate, (a) of Eq. (53). The equilibrium constant is then determined from the partition function. The assumption that the detailed paths of the atomic states are the same in the forward and backward directions (probably not a justified assumption, see Biondi[42]) permits Lin and Teare[4] to write the dissociative recombination rate corresponding to the reverse reaction (53a) as:

$$k_r = \frac{K^{EQ.}}{k_f} \tag{54}$$

where k_f is the forward rate constant

k_r is the reverse rate constant

K^{EQ} is the equilibrium constant.

For $T_e < 10^4$ °K, Lin and Teare arrive at a dissociative reaction rate

$$k_r^{(a)} = 3 \times 10^{-3}\, T_e^{-3/2} \quad cm^3/sec \ . \tag{55}$$

The rather fast rate for dissociative recombination predicts that $k_r^{(a)} = 6\times10^{-7}cm^3/sec$ at $T = 300°$, which appears to agree with the rate coefficient of 2×10^{-6} deduced by Doering and Mahan[43] from photolysis experiments of NO. This piece of experimental data, together with Sugden's[44] observation at flame temperatures, forms the basis of Lin and Teare's acceptance of Eq. (55) as the correct expression for the dissociative recombination coefficient for NO^+. Unfortunately this particular choice of temperature dependence is unrealistic on physical grounds. This may be seen by writing Eq. (52) in the form:

$$k = 4\pi\left(\frac{m}{2\pi kT_e}\right)^{3/2}\int_0^\infty \sigma(v)\, v^3 \exp\left(\frac{-mv^2}{2kT_e}\right) dv$$

where the electron velocity distribution function has been assumed Maxwellian. It may be noted that Eq. (52) and Eq. (55) are compatible only if the velocity-dependent cross section assumes the form:

$$\sigma(v) = \frac{\delta(v)}{v^3}$$

which is not plausible from a physical point of view, especially since the cross section must assume a finite value as the electron velocity approaches zero.

In the present analysis an accurate knowledge of the dissociative recombination rates corresponding to the reverse reactions, (a), (b), and (c) of Eq. (53), is necessary not only for the computation of the steady-state electron density achieved in the re-entry plasma sheath under the influence of high-power electromagnetic waves in the microwave range, but also the electron temperature must be determined from an energy balance equation by a procedure to be outlined in Section 2. 3. 2. The terms in the energy balance equation which represent energy loss of the electron gas due to dissociative recombination involve derivatives of the recombination rate coefficients with respect to electron temperature. The $T_e^{-3/2}$ temperature dependence for the dissociative recombination coefficients of the reverse reactions (53a, b, c), as quoted by Lin and Teare,[4] predict a zero energy loss by the electron gas on dissociative recombination; a result which is not realistic from a physical point of view. Since there is a paucity of experimental data on dissociative recombination coefficients over extended temperature ranges, the procedure to be adopted in this report will consist of assuming a functional form for the associative ionization rate [Eq. (53a)], and then making a least-squares fit to the experimental data of Lin's determination of the associative ionization cross section for O_2 - N_2 mixtures.[45] The dissociative recombination rate will then be determined from Eq. (54) (Section 2. 3. 2).

The recombination of electrons with the positive atomic ions O^+ and N^+ will proceed primarily by the process of three-body recombinations. Radiative recombination will be neglected. The concentrations of O^+ and N^+ become appreciable at distances of ten or more mean free paths behind the shock front (see Lin and Teare[4]). In the absence of electromagnetic field, the chief mechanisms for production of N^+ and O^+ ions are atom-atom and atom-molecule collisions:

$$O + M \rightarrow M + O^+ + e \tag{56a}$$

$$N + M \rightarrow M + N^+ + e \quad . \tag{56b}$$

Unfortunately there is at present no reliable experimental information on the rates of reactions of Eqs. (56a) and (56b). Even though Lin and Teare[4] indicate that estimates of the three-body electron-ion recombination coefficients based on the classical Thomson[36] theory yield rates that seem to be several orders of magnitude too large, calculations based on the Thomson theory will be utilized in this report for want of more accurate information.

Massey and Burhop[7] have derived the three-body electron-ion recombination coefficient by computing the total probability that an electron will suffer a collision with a neutral, lose most of its kinetic energy, and simultaneously remain within a distance 'r_o' of a positive ion. 'r_o' is the maximum distance of electron-ion separation for which the two particles can describe a closed orbit:

$$r_o = \frac{2e^2}{mv^2} \quad , \tag{56}$$

where v = electron velocity, which is much greater than the ion velocity. The Thomson cross section for three-body electron-ion recombination is:

$$\sigma_R(v) = \frac{4\pi}{3} r_o^3 G(v) \frac{1}{\lambda(v)} \quad , \tag{57}$$

where $\lambda(v)$ = mean free path for electron-neutral collisions. Using the fact that the electron-neutral collision frequency $\nu(v)$ is given by $\nu(v) = v/\lambda(v)$, the rate of disappearance of electrons due to three-body electron-ion recombination is given by:

$$-\left(\frac{dN_e}{dt}\right)_i = \int_o^\infty B_o^{\text{recomb.}} \; 4\pi v^2 dv$$

$$= N_i \int_0^\infty f_o \left(\frac{4\pi}{3}\right)\left(\frac{2e^2}{mv^2}\right)^3 \sum_j G_j(v)\, \nu_j(v) \cdot 4\pi v^2 dv$$

$$= N_i N_e \left(\frac{1}{mkT_e}\right)^{3/2} e^6 \frac{32}{3} \sqrt{2\pi} \cdot \int_0^\infty [\Sigma G_j(v)\nu_j(v)] \frac{1}{v^4} \exp\left(\frac{-mv^2}{2kT_e}\right) dv, \tag{58}$$

where $G_j(v)$ is the fractional energy loss of an electron in collisions with the j^{th} neutral constituent

and $\nu_j(v)$ is the electron collision frequency for collisions with the j^{th} neutral constituent.

Eschenroeder[37] has calculated the three-body electron-ion recombination coefficient according to the Thomson theory, but has misinterpreted G as the fractional energy loss of an electron for electron-ion collisions, instead of electron-neutral collisions. For a given neutral species, $G(v)$ may be computed by solving the integral Eq. (26). The collision frequency $\nu(v)$ may be computed for each species by using the total cross section vs electron energy curves as tabulated by Shkarofsky et al.[32] The relative concentration of each neutral specie may be found from Figure 4.1 of the report by Bachynski et al.[35] It should be noted that Eq. (58) is a generalization of the Thomson three-body electron-ion recombination formula for the case where the electron temperature is not necessarily equal to the gas temperature and the role of the third body is played by a mixture of neutral constituents.

2.2 Solutions of the Zero Order and First Spherical Harmonic Components of the Boltzmann Equation

The zero-order and first spherical harmonic components of the collision integral which appear on the right-hand side of Eq. (18a) and (18b) represent the sum of all types of electron collisions:

$$B_o = B_o^{el} + B_o^{inel} + B_o^{ion} + B_o^{ee} \tag{59a}$$

$$B_1 = B_1^{el} + B_1^{inel} + B_1^{ion} + B_1^{ee} \quad . \tag{59b}$$

Substituting Eq. (59a) into (18a) and (59b) into (18b) yields:

$$\frac{\partial f_o}{\partial t} + \frac{eE}{3mv^2} \frac{\partial}{\partial v}(v^2 f_1) = \frac{1}{2v^2} \frac{\partial}{\partial v}\left\{\left(\frac{2m}{M}\nu_{ion} + \Sigma G_j \nu_j\right) \cdot v^2 \left[\frac{kT}{m} \frac{\partial f_o}{\partial v} + v f_o\right]\right\}$$

$$+ \frac{1}{v^2} \frac{\partial}{\partial v}\left\{v^2 \left[A_1(v) v f_o + A_2(v) \frac{\partial f_o}{\partial v}\right]\right\} + B_o^{ioniz} + B_o^{recomb.} \tag{60a}$$

$$\frac{\partial \vec{f}_1}{\partial t} + \frac{e\vec{E}}{m} \frac{\partial f_o}{\partial v} = -(\nu + \nu_{ion})\vec{f}_1 - \frac{\nu_{ee}}{\omega}(\nu + \nu_{ion})\vec{f}_1 \tag{60b}$$

where $\nu = \Sigma N_j \int v\, \sigma_j(\theta, v)(1-\cos\theta)\, d^2\Omega$

N_j is the particle density of the j^{th} neutral constituent and σ_j is the cross section for elastic and inelastic scattering.

The term

$$\frac{1}{v^2}\frac{\partial}{\partial v}\left\{ v^2 \left[A_1(v)\, vf_o + A_2(v) \frac{\partial f_o}{\partial v}\right]\right\}$$

is of the order $(\nu_{ee} f_o)$. Hence, the form of the function f_o depends upon the relative magnitude of the parameters ν_{ee} and $G\nu$.

The condition $\nu_{ee} \gg G\nu$ is sufficient to guarantee that f_o be Maxwellian, even when ionizing and recombination collisions are present for the conditions considered in this analysis. These conditions include the facts that (1) the plasma is less than 0.1 per cent ionized, and (2) the fractional energy loss of an electron per collision G is always less than or equal to 10^{-2}.

Ginzburg and Gurevich[8] define a strongly ionized plasma as a plasma for which the condition $\nu_{ee} \gg G\nu$ pertains; and a weakly ionized plasma as one in which the condition $\nu_{ee} \ll G\nu$ is satisfied. Under the assumption that B_o^{ioniz} and B_o^{recomb} are zero, the Boltzmann equation for the isotropic part of the distribution function [Eq. (60a)] may be solved for the general case ($\nu_{ee} \approx G\nu$) by the method of successive approximations. As noted by Ginzburg and Gurevich,[8] this method converges rapidly because variations in f_o cause only small changes in the parameters A_1 and A_2 [defined by Eq. (41)]. In the first approximation, a Maxwellian distribution function

$$f_{oo} = N_e \left(\frac{m}{2\pi k T_e}\right)^{3/2} \exp\left(\frac{-mv^2}{2kT_e}\right) \tag{61}$$

is substituted into Eq. (60a). By a method to be described in detail in Section 2.3, the electron particle density is found by taking the zero-order velocity moment of Eq. (60a) and the electron temperature is obtained by taking the second velocity moment. Then, the second approximation to Eq. (60a) may be written:

$$f_{01} = C \exp\left\{-\int_o^v \frac{v\,dv \left[\Sigma G_j \nu_j + \frac{2m}{M}\nu_{ion} + 2A_1^o\right]}{\left[kT/m\left(\Sigma G_j \nu_j + \frac{2m}{M}\nu_{ion}\right) + \frac{2e^2E^2\nu}{3m^2(\omega^2+\nu^2)} + 2A_2^o\right]}\right\} \tag{62}$$

where $\nu = (\nu_{en} + \nu_{ion})(1 + \nu_{ee}/\omega)$ and C is a normalizing constant. Here,

$$A_2^o = \frac{kT_e}{m} A_1^o = \frac{kT}{m} \nu_{ee}(v) \left[\phi(y) - \frac{2}{\sqrt{\pi}} y \exp(-y^2)\right]$$

where

$$\phi(y) = \frac{2}{\sqrt{\pi}} \int_o^y \exp(-\mu^2)\, d\mu \qquad \text{(error function)}$$

$$y \equiv \nu \left(\frac{m}{2kT_e}\right)^{1/2} \quad .$$

In the derivation of Eq. (62), it has been assumed that $\partial f_{oo}/\partial t = 0$ (steady-state case, $\omega >> G_j\nu_j$). An expression for f_1 has been obtained from Eq. (60b), where f_1 has been assumed proportional to $e^{j\omega t}$. This result was first obtained in a somewhat different manner by Cahn[19] and later by Ginzburg and Gurevich.[8]

Equation (62) is the most general expression for the electron velocity distribution function in a partially-ionized multicomponent plasma (in which ionizing and recombination collisions are neglected). For positions on a blunt-nosed re-entry vehicle's surface at distances of two or three nose radii back from the tip, particularly for vehicles traveling slower than Mach 16 at 200,000 ft, the temperatures and pressures behind the shock are such that $\nu_{ee} \approx G\nu$. For this case, Eq. (62) should be used in calculating the electrical transport properties (conductivity) of the plasma. However, in this report, only regions in and near the stagnation region of re-entry vehicles at 200,000 ft and traveling at about Mach 16 or greater will be considered, for which $\nu_{ee} >> G\nu$. In this case it may be shown that Eq. (62) reduces to Maxwellian form ($f_{o1} = f_{oo}$). For the other extreme ($\nu_{ee} << G\nu$) it may be shown that the general expression for the isotropic part of the distribution function reduces to the Margenau form [Eq. (9)]. Only for the case $\nu_{ee} >> G\nu$ is the computation of the changes in electron temperature, electron density, and collision frequency due to the presence of an electromagnetic field relatively straightforward, because the ionization rates, recombination rates and fractional energy loss parameters are known directly in terms of electron temperature.

The solution of the equation for the first spherical harmonic component of the electron velocity distribution function [Eq. (60b)] may be written:

$$\vec{f}_1 = -\frac{e\vec{E}}{m}\left[\frac{1}{j\omega + (\nu + \nu_{ion})(1 + \nu_{ee}/\omega)}\right]\frac{\partial f_o}{\partial v} \quad . \tag{63}$$

Utilizing Eqs. (63) and (10), it is possible to find an expression for the AC conductivity of a partially-ionized plasma in terms of the isotropic part of the electron distribution function (assumed to be Maxwellian in this case), the frequency of the applied field (ω), the total electron-neutral, electron-ion, and interelectron collision frequencies, and the electron density:

$$\sigma = \frac{e}{\vec{E}} \int \vec{v} f d^3v = \frac{e}{\vec{E}} \int \vec{v}\, \frac{(\vec{f}_1 \cdot \vec{E})}{v}\, d^3v$$

$$= \frac{e}{\vec{E}} \int \vec{v}(\vec{f}_1 \cdot \vec{v})\, 2\pi \sin\theta d\theta v dv = \frac{e}{E}\left(\frac{4\pi}{3}\right)\int_0^\infty f_1 v^3 dv \qquad (64)$$

where $\vec{f}_1$ lies along the $\vec{E}$ field, taken along the Z-axis. Substituting Eq. (63) into Eq. (64) yields:

$$\sigma = \frac{8e^2N_e}{3\sqrt{\pi}\, m}\left[\int_0^\infty \frac{\nu(u)(1+\nu_{ee}/\omega)\, u^4 e^{-u^2} du}{\omega^2 + \nu^2 (1+\nu_{ee}/\omega)^2} - j\omega\int_0^\infty \frac{u^4 e^{-u^2} du}{\omega^2 + \nu^2 (1+\nu_{ee}/\omega)^2}\right], \qquad (65)$$

where

$$\nu(v) = \Sigma\, N_j\, \sigma_j\, v + \nu_{ion}$$

$$u \equiv \sqrt{\frac{m}{2kT_e}}\; v$$

$$\nu_{ee}(v) = 2\pi N_e \frac{e^4}{m^2 v^3}\, \ln\left(\frac{\ell_D^2\, m^2 v^4}{e^4}\right).$$

The conductivity is related to the dielectric constant (MKS) through the relation:

$$K = 1 + \frac{\sigma}{j\omega\epsilon_o} \quad . \qquad (66)$$

Typical parameters for regions in and near the stagnation region of blunt-nosed re-entry vehicles at 200,000 ft at Mach 16 are:

$N_e \approx 5 \times 10^{12}$ elec/sec

$\omega_p \approx 10^{11}$ rad/sec

$$\nu \quad \approx 5 \times 10^9 \text{ coll/sec}$$

$$\nu_{ee} \quad \approx 10^9 \text{ coll/sec}$$

$$G \quad \approx 5 \times 10^{-3}$$

$$\nu_{ee} \quad \approx 50G\nu \ .$$

For the case of a monochromatic plane wave incident on a slab of plasma (having the characteristics of the re-entry sheath), most of the electromagnetic energy will be coupled into the plasma only if the frequency of the plane wave (ω) lies within a factor of two or three times greater than or less than the plasma frequency ω_p: $\omega_p/3 \lesssim \omega \lesssim 3\omega_p$. Otherwise, most (90 per cent or more) of the energy will be either-reflected from the first interface or transmitted through the slab. However, since the nonlinear properties of the plasma sheath (changes in electron density and collision frequency) will be manifested only when there is appreciable coupling between the electromagnetic wave and the plasma, the following conditions will pertain to the range of parameters where maximum nonlinearity occurs:

$$\omega^2 >> \nu^2$$

and $\qquad \nu_{ee}/\omega << 1$.

This implies that the dielectric constant may be written in the form:

$$K = 1 - \frac{\omega_p^2}{\omega^2} - j\left(\frac{\nu_{EFF}}{\omega}\right)\frac{\omega_p^2}{\omega^2} \tag{67}$$

where ν_{EFF} is defined by Eq. (2):

$$\nu_{EFF} = \frac{\sqrt{2}}{3\sqrt{\pi}}\left(\frac{m}{kT_e}\right)^{5/2} \int \nu(v)\, v^4 \exp\left(-\frac{mv^2}{2kT_e}\right) dv$$

and $\qquad \omega_p^2 = \dfrac{N_e e^2}{m\epsilon_o}$.

Hence, for the high-frequency case ($\omega^2 \gg \nu^2$) the total effective collision frequency is the sum of the effective collision frequency between electrons and neutrals and the effective collision frequency between electrons and ions:

$$\nu_{EFF} = (\nu_{en})_{EFF} + (\nu_{ion})_{EFF}$$

where $(\nu_{ion})_{EFF}$ is given by Eq. (33):

$$(\nu_{ion})_{EFF} = \frac{2}{3}\sqrt{\frac{8\pi}{m}}\ \frac{N_e e^2}{(kT_e)^{3/2}}\ \ell n\left(\frac{kT_e \ell_D}{e^2}\right) .$$

Thus Eq. (67) for the dielectric constant will form the basis for the computations of the nonlinear transmission characteristics of the re-entry plasma sheath. The parameter $(\nu_{en})_{EFF}$ may be calculated from a knowledge of the electron temperature T_e, electron density N_e, and the concentrations of the neutral constituents N_j, together with their respective cross sections σ_j. The concentrations of the neutral constituents are calculated from the graphs of the total relative density (ρ/ρ_o) profile and gas temperature (T) profile behind the shock front found in the AFCRL report by Rotman and Meltz[11] and the curves of the relative concentrations of the neutral constituents O_2, N_2, O, N, and NO plotted as a function of ρ/ρ_o and T located in the report by Bachynski et al.[38] (Figure 4.1). The total cross sections for electron scattering $\sigma_j(v)$ may be found in the report by Shkarofsky et al.[32] for the various constituents O_2, N_2, N, O, and NO plotted as a function of electron energy ($\frac{1}{2}$ mv^2). The procedure for the determination of T_e and N_e at each point in the plasma slab will be discussed in Section 2.3.

2.3 Particle Conservation and Energy Conservation: The Zero and Second Velocity Moments of the Boltzmann Equation

2.3.1 THE STEADY-STATE ELECTRON CONCENTRATION

When $\nu_{ee} \gg G\nu$, f_o is Maxwellian and an expression for the electron density may be obtained by multiplying both sides of Eq. (60a) by $4\pi v^2 dv$ and integrating. The following expressions are the only nonvanishing terms which result from the integration:

$$\frac{\partial}{\partial t}\int f_o\, 4\pi v^2 dv = \frac{\partial N_e}{\partial t} \tag{68a}$$

$$\sum \int_0^\infty B_0^{\text{ioniz}}\, 4\pi v^2 dv = 4\pi \sum N_j \int_0^\infty f_0\, \sigma_{\text{ion}_j}\, v^3 dv$$

$$= N_e \left(\frac{8kT_e}{\pi m}\right)^{1/2} \sum N_j \exp\left(\frac{-w_{ij}}{kT_e}\right) \cdot \pi a_0^2 \tag{68b}$$

$$\sum \int_0^\infty B_0^{\text{dissoc. recomb.}}\, 4\pi v^2 dv = 4\pi \sum N_i \int_0^\infty f_0\, \sigma_{DR_i}(v)\, v^3 dv$$

$$= 4\pi N_e \sum N_i \left(\frac{m}{2\pi kT_e}\right)^{3/2} \int_0^\infty \sigma_{DR_i} \exp\left(\frac{-mv^2}{2kT_e}\right) v^3 dv$$

$$= N_e \sum N_i\, k_i(T_e) \tag{68c}$$

$$\sum \int_0^\infty B_0^{\text{three-body recomb.}}\, 4\pi v^2 dv = N_e \left(\frac{32}{3}\right) \sqrt{2\pi}\, e^6 \left(\frac{1}{mkT_e}\right)^{3/2} \tag{68d}$$

$$\cdot (\Sigma N_i) \int_0^\infty \left[\sum_j G_j(v)\, \nu_j(v)\right] \frac{1}{v^4} \exp\left(\frac{-mv^2}{2kT_e}\right) dv.$$

Equations (68b), (68c), and (68d) have been derived in Section 2. 1. 5. Since, in the steady state, the electron density is equal to the sum of the ion densities ($N_e = \Sigma N_i$), the zero-order moment of the Boltzmann equation for the isotropic part of the distribution function may be written:

$$\frac{dN_e}{dt} = \sum \frac{dN_i}{dt} = N_e \left(\frac{8kT_e}{\pi m}\right)^{1/2} \sum N_j \exp\left(\frac{-w_{ij}}{kT_e}\right) (\pi a_0^2) - N_e \sum N_i\, k_i(T_e) \tag{69}$$

$$- N_e \left(\frac{32}{3}\right) \sqrt{2\pi}\; e^6 \left(\frac{1}{mkT_e}\right)^{3/2} (\Sigma N_i) \cdot \int_0^\infty (\Sigma G_j \nu_j) \frac{1}{v^4} \exp\left(\frac{-mv^2}{2kT_e}\right) dv.$$

It should be noted that Eq. (69) does not include the effects of electron production resulting from 'external' agencies, such as photoionization, and neutral-neutral impact.

On the basis of Lin and Teare's investigation, the examination of the many possible electron production and recombination mechanisms (such as atom-atom and atom-molecule impact, electron-neutral and electron-ion impact, photoionization, electron-neutral attachment, dissociative recombination, and three-body recombination) indicates that the following reactions are the predominate processes in the presence of an electromagnetic field which raises the electron temperature from 0.5 to 2 ev:

$$
\begin{aligned}
&N + O \rightarrow NO^+ + e && (1)\\
&N + N \rightarrow N_2^+ + e && (2)\\
&O + O \rightarrow O_2^+ + e && (3)\\
&X + N \rightarrow N^+ + X + e && (4)\\
&X + O \rightarrow O^+ + X + e && (5)\\
&NO + e \rightarrow NO^+ + 2e && (6)\\
&O_2 + e \rightarrow O_2^+ + 2e && (7)\\
&N_2 + e \rightarrow N_2^+ + 2e && (8)\\
&N + e \rightarrow N^+ + 2e && (9)\\
&O + e \rightarrow O^+ + 2e && (10)\\
&N_e = N^+ + O^+ + N_2^+ + O_2^+ + NO^+ \ . && (11)
\end{aligned}
\tag{70}
$$

Then, the rate of production for each ionic specie is given by the expressions:

$$
\begin{aligned}
\frac{d(NO^+)}{dt} &= k_{f_1}(N)(O) + k_{f_6}(NO)(N_e) - k_{r_1}(NO^+)(N_e)\\
\frac{d(N_2^+)}{dt} &= k_{f_2}(N)^2 + k_{f_8}(N_2)(N_e) - k_{r_2}(N_2^+)(N_e)\\
\frac{d(O_2^+)}{dt} &= k_{f_3}(O)^2 + k_{f_7}(O_2)(N_e) - k_{r_3}(O_2^+)(N_e)\\
\frac{d(O^+)}{dt} &= k_{f_5}(X)(O) + k_{f_{10}}(O)(N_e) - k_{r_5}(O^+)(X)(N_e)\\
\frac{d(N^+)}{dt} &= k_{f_4}(N)(X) + k_{f_9}(N)(N_e) - k_{r_4}(N^+)(X)(N_e) \ .
\end{aligned}
\tag{71}
$$

Using particle conservation, charge neutrality, and the steady-state condition

$$\left(\frac{dN_e}{dt} = \frac{d}{dt}\Sigma N_i = 0\right) ,$$

an equation for the steady-state electron density may be obtained from expressions (71):

$$N_e = \frac{1}{2} \sum_j \frac{N_j k_{f_j}^{ion}}{k_{r_j}} + \frac{1}{2} \left[\left(\sum \frac{N_j k_{f_j}^{ion}}{k_{r_j}} \right)^2 + 4 \left(\sum \frac{N_j' k_{f_j}^{N}}{k_{r_j}} \right) \right]^{1/2} \tag{72}$$

where $k_{f_j}^{ion}$ refers to the rate of ionization of neutrals by electron impact.

$$N_j \, k_{f_j}^{ion} = \left(\frac{8kT_e}{\pi m}\right)^{1/2} N_j \exp\left(\frac{-w_i}{kT_e}\right)(\pi a_o^2) \quad ,$$

where N_j is the particle concentration of the j^{th} neutral constituent. The relative concentrations of N, O, N_2, O_2, and NO as a function of ρ/ρ_o and gas temperature T are taken from Figure 4. 1 of Bachynski et al.[38] The ionization potentials of these constituents are given in Table 2:

TABLE 2

	W_i (ev)
NO	9. 5
O_2	12. 5
N_2	15. 7
N	14. 5
O	13. 6

The relative density (ρ/ρ_o) and temperature profiles along the line located at 45° with respect to the axis of the hemisphere cylinder are given in Rotman and Meltz.[11] These profiles were determined for equilibrium flow conditions in which the boundary layer is neglected. The nonlinear electromagnetic transmission characteristics of the re-entry sheath computed in this report will be based upon the calculations presented in the report by Rotman and Meltz[11] for the line located at 45° with respect to the axis of the hemisphere cylinder.

The forward rate for reaction (1) will be obtained by taking a theoretically derived expression for the associative ionization cross section and making a least-squares fit of this functional form to the experimental data of Lin.[45] This procedure will be explained in detail in Section 2. 3. 2 in connection with the energy loss of the electron gas in dissociative recombination. This will give the expression for k_{f_1}. The expression for the dissociative recombination coefficient k_{r_1} will be obtained through the equilibrium constant

$$k_{r_1} = \frac{k_{f_1}}{K_1^{EQ}} \quad .$$

where $K_1^{EQ} = (1.4 \times 10^{-8}T + 1.2 \times 10^{-12}T^2 + 1.4 \times 10^{-16}T^3) \exp(-32,500/T)$.

For want of better experimental data, it will be assumed that the dissociative recombination coefficients of the reverse reactions 1, 2, and 3 will all be equal:

$$k_{r_1} = k_{r_2} = k_{r_3} \quad .$$

Then the forward-rate coefficients for the associative ionization reactions (2) and (3) will be determined from their respective equilibrium constants:

$$k_{f_2} = (K_2^{EQ})\, k_{r_1}$$

and

$$k_{f_3} = (K_3^{EQ})\, k_{r_1}$$

where the equilibrium constants for reactions (2) and (3) have been obtained from the partition functions (Lin and Teare[4]):

$$K_2^{EQ} = (3 \times 10^{-8}T + 4 \times 10^{-12}T^2 + 10^{-15}T^3 - 3 \times 10^{-20}T^4) \cdot \exp(-67,300/T)$$

$$K_3^{EQ} = (1.6 \times 10^{-8}T + 1.2 \times 10^{-12}T^2 + 3.5 \times 10^{-16}T^3) \cdot \exp(-80,100/T) \quad .$$

The forward rates for reactions (4) and (5) were estimated by the simple collision theory presented by Bortner[39] and tabulated in the report by Atallah:[40]

$(N_j^{'}\ k_{f_j}^N)$

(1) $(N)(O)\ k_{f_1}$

(2) $(N)(N)\ K_2^{EQ} k_{r_1}$

(3) $(O)(O)\ K_3^{EQ} k_{r_1}$

(4) $(X)(N)\ 4.86 \times 10^{-12} T^{\frac{1}{2}} \exp(-333,000/RT)$

(5) $(X)(O)\ 4.62 \times 10^{-12} T^{\frac{1}{2}} \exp(-313,000/RT)$.

Lin and Teare[4] have pointed out that there is no direct experimental data available in the literature for the forward rates (4) and (5). The backward rates may be computed on the basis of the Thomson theory. It should be noted that the terms $(N_j^{'})k_{f_j}^N$, which appear in expressions (67) for the electron density (N_e), all play the role of a constant. These terms are constant for they are functions only of the neutral concentrations and the gas temperature (and not electron temperature). Both the gas temperature and the concentrations of the neutral constituents are assumed to be unchanged in the presence of the electromagnetic field. This is a good assumption because the degree of ionization of the plasma sheath is between 0.1 and 0.01 per cent or less ($\rho/\rho_o = 10^{-3}$ to 10^{-2}, $N_e \approx 10^{12}$ or 10^{13} elec/cm^3).

The terms k_{r_j} in Eq. (72) refer to the backward rates of reactions 1 through 5. The backward rates 1 through 3 refer to dissociative recombination. Since the temperature dependence $(T_e^{-3/2})$ of the dissociative recombination rates corresponding to the backward rates 1 through 3 as given by Lin and Teare is inadequate for the purposes of the present analysis because it predicts zero-energy loss of the electron gas on dissociative recombination, the dissociative recombination rate for the reverse of reaction 1 will be determined by making a least-squares computation to be explained in Section 2.3.2. The dissociative recombination coefficients corresponding to the reverse reactions 1 through 3 will all be assumed equal. The backward rates of reactions 4 and 5 refer to the Thomson three-body recombination coefficient, generalised to the case of a multicomponent plasma where the electron temperature is not equal to the gas temperature [Eq. (58)]:

$$k_{r_4} = k_{r_5} = \left(\frac{1}{mkT_e}\right)^{3/2} e^6 \left(\frac{32}{3}\right) \sqrt{2\pi} \int_0^\infty \left[\Sigma G_j(v)\nu_j(v)\right] \frac{1}{v^4} \exp\left(\frac{-mv^2}{2kT_e}\right) dv,$$

$G_j(v)$ is the fractional energy loss of an electron per collision with the j^{th} neutral constituent. The parameter G is usually determined as a function of electron temperature (T_e), and is listed in Table 1 for various gases. Notice that the three-body ion-electron recombination rates [Eq. (58)] do not have to be calculated by first finding G(v). This would involve solving the integral Eq. (26) for G(v) for each constituent, and then substituting the result into Eq. (58). A much more elegant technique consists in taking the right-hand side of Eq. (26) (known for each constituent) and integrating four times with respect to electron temperature. This will yield the correct form for the generalized Thomson three-body recombination rate:

$$k_{r_4} = k_{r_5} = T_e^{-3/2} \left(\frac{2\pi e^6}{k^3}\right) \sum_j \int_0^{T_e} \frac{1}{s^2} \int_0^{s} \frac{1}{u^2} \int_0^{u} \frac{1}{v^2} \int_0^{v} W^{1/2}\, G_j(w)\, \nu_{j_{EFF}}(w)\, dw\, dv\, du\, ds.$$

The effect of charge exchange has been neglected in the derivation of Eq. (72) for the electron density. This mechanism appears to have a relatively small effect on the total electron concentration. Under the action of the charge exchange mechanism, the relative concentrations of the various ionic species may change. This may produce a change in the total electron concentration, because the electron-ion recombination rates are specie dependent.

2.3.2 THE ENERGY BALANCE EQUATION

When the relaxation time for energy transfer between the electron gas and the neutral gas is much greater than the period of a monochromatic electromagnetic wave impressed upon the plasma ($\tau_{EN} = \frac{1}{G\nu} >> T = 2\pi/\omega$), then the electron gas will no longer be in thermodynamic equilibrium with the neutral gas but will achieve a temperature that depends upon the mean square value of the impressed field. In the steady state the temperature of the electron gas is determined from an energy balance equation. This energy balance equation, obtained by taking the second velocity moment of the Boltzmann equation for the isotropic part of the distribution function [Eq. (60a)], is an expression of the fact that in the steady state the energy acquired by the electron gas due to ohmic heating by the electromagnetic field is equal to the various electron energy loss processes. The electron energy loss processes for a multicomponent plasma include:

(1) Elastic electron-neutral collisions
(2) Elastic electron-ion collisions
(3) Electron-molecule collisions which excite rotational, vibrational, and electronic levels
(4) Ionizing collisions by electron impact on neutrals
(5) Ion-electron dissociative and three-body recombination collisions.

All of these processes will be taken into account in the computation of the steady-state electron temperature. Other electron energy loss processes for a particular point in the plane-layered plasma slab that may be neglected in the energy balance equation include:

(6) Heat flow due to electron-electron collisions

$$[\, K_{ee}(T_e)\, \nabla^2 T_e - K_{ee}(T)\, \nabla^2 T \,]$$

(7) Heat flow due to electron-neutral collisions

$$[\, K_{en}(T_e)\, \nabla^2 T_e - K_{en}(T)\, \nabla^2 T \,]$$

(8) Heat flow due to particle diffusion

$$\left[\frac{64k^2 T_e}{9\pi m \nu_{EFF}}\, \nabla N_e \cdot \nabla T_e - \frac{64k^2 T}{9\pi m \nu_{EFF}}\, \nabla N_e \cdot \nabla T\right].$$

These last three electron energy loss processes will be shown to be negligible in Section 2.4.

There are several important facts that bear careful consideration. First, the term $(2m/M)\nu_{ion}$ in Eq. (60a) refers to elastic collisions between electrons and a single ion specie of 'average' mass M. The general term for a plasma containing many ionic species would be written:

$$\sum \frac{2m}{M_i}\left[4\pi N_i \frac{e^4}{m^2 v^3}\, \ell n\left(\frac{\ell_D m v^2}{e^2}\right)\right]$$

where M_i is the mass and N_i the particle concentration of the i^{th} ionic specie. The approximation consists of replacing M_i by an average mass M and $\Sigma N_i = N_e$ (steady state). This results in a great simplification of the equations while introducing only a small error.

Another important point involves the precise balancing of all energy gain and loss processes of the electron gas. It was previously mentioned that electron production mechanisms due to external agencies, such as neutral-neutral impact, are not included in the collision integral terms appearing on the right-hand side of the Boltzmann equation [Eq. (60a)]. Processes such as associative ionization corresponding to the forward reactions (1) through (3) and ionization by neutral-atom impact corresponding to the forward reactions (4) and (5) have been taken into account in the expression for the steady-state electron concentration, Eq. (72).

Moreover the forward reactions (1) through (5) constitute a net energy gain by the electron gas which is not changed by the presence of an electromagnetic field since the forward reactions do not depend upon the electron temperature. However the dissociative and three-body recombination processes corresponding to the backward reactions (1) through (5) constitute an energy loss to the electron gas which does depend upon the electron temperature.

The consideration of the energy gain and loss terms due to ionizing collisions by electron impact corresponding to the forward reactions (6) through (10) is more subtle. The creation of a new electron by electron impact on a neutral constituent represents a gain of energy by the electron gas, which may be computed by taking the second velocity moment of the collision integral for ionizing collisions. The impacting electron, which is responsible for the ionization, suffers a loss of energy, which is included in the parameter G. This is because G represents the total fractional energy loss of an electron due to all electron-neutral collision processes-elastic electron-neutral collisions, collisions which excite vibrational, rotational, and electronic levels of molecules, and ionizing collisions. In the absence of an electromagnetic field, when the electron gas is in thermodynamic equilibrium with the neutral gas, each of these numerous processes occurs at a rate such that there is no net gain or loss of energy by the electron gas. The application of an electromagnetic field to the ionized flow field results in an elevation of the electron temperature with a consequent change in some of the rate processes. The energy lost by the electromagnetic field to the plasma is dissipated only by those energy loss processes over and above what is suffered by the electron gas when in equilibrium with the neutral gas. This implies that each energy gain or loss process of the electron gas must be represented in the energy balance equation as the difference between two terms. One term refers to the energy gained or lost by the electron gas in the presence of the electromagnetic field at the elevated electron temperature (T_e) and electron density $N_e(T_e)$, whereas the second term refers to the energy gained or lost by the electron gas when in equilibrium with the neutral gas at temperature (T) and electron density $N_e(T)$. Thus, it may be seen that each term in the energy balance equation refers to an energy gain or loss by the electron gas corresponding to the difference between two states of the electron gas: one state of the electron gas is at the temperature T_e and electron density $N_e(T_e)$ in the presence of the electromagnetic field, the other corresponds to the equilibrium state at temperature (T) and electron density $N_e(T)$. The appropriate terms in the energy balance equation are:

The time rate of change of the total kinetic energy in the electron gas:

$$\frac{d}{dt}\left(2\pi m\int_0^\infty v^4 f_0 dv\right) = \frac{3}{2}\ k\ \frac{d}{dt}\ [N_e(T_e)T_e]\ . \tag{74a}$$

The ohmic heating of the electron gas by the electromagnetic field:

$$\frac{4\pi e}{3} \vec{E} \cdot \int_0^\infty v^3 \vec{f}_1 dv = \sigma_r E^2 = \frac{N_e(T_e)e^2 \nu_{EFF} E^2}{m\omega^2} \tag{74b}$$

where

$$\nu_{EFF} = (\nu_{ion})_{EFF} + \frac{\sqrt{2}}{3\sqrt{\pi}} \left(\frac{m}{kT_e}\right)^{5/2} \int \sum \nu_j v^4 \exp\left(\frac{-mv^2}{2kT_e}\right) dv .$$

Electron energy loss due to elastic electron-ion collisions and inelastic electron-neutral collisions.

$$2\pi m \int_0^\infty \left[\frac{2m}{M} \nu_{ion} + \Sigma G_j(v)\nu_j(v)\right] v^3 \left(\frac{kT}{m} \frac{\partial f_o}{\partial v} + \nu f_o\right) dv =$$

$$= \frac{3}{2} \left(\frac{2m}{M}\right) N(T_e)\, k(T_e - T)(\nu_{ion})_{EFF} + \frac{3}{2} \Sigma G_j(T_e) \left[\nu_{EFF}(T_e)\right]_j k N_e(T_e)(T_e - T) \tag{74c}$$

where

$$(\nu_{ion})_{EFF} = \frac{2}{3} \sqrt{\frac{8\pi}{m}}\; \frac{N_e(T_e)e^2}{(kT_e)^{3/2}}\; \ell n\left(\frac{kT_e \ell_D}{e^2}\right) .$$

Energy gain of the electron gas due to creation of new electrons by ionizing electron-neutral collisions.

$$\sum (2\pi m) \int_0^\infty v^4 \left[B_o^{ioniz}(T_e) - B_o^{ioniz}(T)\right] dv$$

$$= 2\pi m \sum N_j \int_0^\infty \left[f_o(T_e) - f_o(T)\right] \sigma_{ion_j} v^5 dv \tag{74d}$$

$$\sum (2\pi m) \int_0^\infty v^4 \left[B_o^{ioniz}(T_e) - B_o^{ioniz}(T)\right] dv = \frac{3}{2} k \sum \int_0^\infty \left[T_e B_o^{ioniz}(T_e) - T\, B_o^{ioniz}(T)\right] \cdot$$

$$\cdot 4\pi v^2 dv + kT_e^2 \frac{d}{dT_e} \sum \int_0^\infty B_o^{ioniz}(T_e)\, 4\pi v^2 dv - kT^2 \frac{d}{dT} \sum \int_0^\infty B_o^{ioniz}(T) 4\pi v^2 dv$$

$$- kT_e^2 \left(\frac{dN_e}{dT_e}\right) \frac{1}{N_e} \sum \int_0^\infty B_o^{ioniz}(T_e)\, 4\pi v^2 dv + kT^2 \left(\frac{dN_e}{dT}\right) \frac{1}{N_e} \sum \int_0^\infty B_o^{ioniz}(T) \cdot$$

$$\cdot 4\pi v^2 dv$$

$$= 2kT_eN_e(T_e)\left(\frac{8kT_e}{\pi m}\right)^{1/2} \sum N_j \exp\left(\frac{-w_{ij}}{kT_e}\right)(\pi a_o^2)$$

$$+ N_e(T_e)\left(\frac{8kT_e}{\pi m}\right)^{1/2} \sum N_j W_{ij} \exp\left(\frac{-w_{ij}}{kT_e}\right)(\pi a_o^2) \tag{74d}$$

$$-2kTN_e(T)\left(\frac{8kT}{\pi m}\right)^{1/2} \sum N_j \exp\left(\frac{-w_{ij}}{kT}\right)(\pi a_o^2)$$

$$-N_e(T)\left(\frac{8kT}{\pi m}\right)^{1/2} \sum N_j W_{ij} \exp\left(\frac{-w_{ij}}{kT}\right)(\pi a_o^2) \quad .$$

Energy loss of the electron gas due to dissociative recombination

$$2\pi m \sum \int_o^\infty \left[B_o^{\text{dissoc. recomb.}}(T_e) - B_o^{\text{dissoc. recomb.}}(T)\right] v^4 dv$$

$$= 2\pi m \sum N_i \int_o^\infty [f_o(T_e) - f_o(T)]\ \sigma_{DR_i}\ v^5 dv$$

$$= \frac{3}{2}\ k\,T_e\ N_e(T_e) \sum N_i(T_e)\ k_i(T_e) + kT_e^2\ N_e(T_e) \sum N_i(T_e)\frac{dk_i(T_e)}{dT_e} \tag{74e}$$

$$-\frac{3}{2}\ kTN_e(T) \sum N_i(T)\ k_i(T) - kT^2\ N_e(T) \sum N_i(T)\ \frac{dk_i(T)}{dt}\ .$$

As mentioned previously, Lin and Teare's[4] choice of $(T_e^{-3/2})$ for the temperature dependence of the dissociative recombination rates (corresponding to the reverse reactions 1, 2, and 3) is unrealistic from a physical point of view because it requires the velocity-dependent cross section for dissociative recombination to assume the form $\sigma(v) = \delta(v)/v^3$, where $\delta(v)$ is the Dirac delta function. A $(T_e^{-3/2})$ temperature dependence of the rate coefficient predicts zero energy loss by the electron gas for dissociative recombination. This implausible result is a consequence of the curve-fitting procedure Lin and Teare have used for the determination of the associative ionization rate corresponding to the forward reaction 1. Lin and Teare have expressed the general functional form of the associative ionization cross section on the basis of the Bates and Massey[46] potential energy curve-crossing model:

$$Q_{E>E_x} = g\pi(R_x a_o)^2 \left[1 - \exp\left(-3.8 \times 10^{-15}\ \frac{\mu^{1/2} R_x}{E^{1/2}\,\tau_{AI}}\right)\right]$$

where E_x equals activation energy and g is the ratio between the statistical weight associated with the initial potential energy curve which leads to the crossing point and the sum of the statistical weights associated with all possible initial potential energy curves of the colliding atomic system. μ is the reduced mass of the colliding system and $R_x a_o$ is the distance between the two centers of mass of the colliding system at the point of crossing. E is the impact energy $E = \frac{1}{2} \mu v^2$ (in electron volts) and τ_{AI} is the autoionization lifetime of the molecular complex. While there is no definite information on the value of the autoionization lifetime of complex atomic systems, the assumption that $\tau_{AI} \ll 10^{-14}$ leads to a velocity dependent cross section expressible in the form:

$$Q_{E > E_x} = AE^{-\frac{1}{2}}$$

where A is a constant. This leads to a velocity-averaged cross section of the form:

$$\bar{Q} = BT^{-3/2} \int_{E_x}^{\infty} Q(E)\, E^{1/2} \exp\left(\frac{-E}{kT}\right) dE$$

$$= aT^{-1/2} \exp\left(\frac{-E_x}{kT}\right) .$$

Lin[45] has measured the rate for the associative ionization reaction 1 in the temperature range 4000° to 5000°K by monitoring both the DC conductivity and the microwave attenuation in shock heated 0.25% O_2 - 99.75% N_2 mixtures. The velocity-averaged associative ionization cross sections for this data are plotted as a function of temperature in Figure 5 of the report by Lin and Teare.[4] In their report the velocity-averaged associative ionization cross section for the extended temperature range 300 < T < 30,000°K is determined by fitting the functional form $Q = AT^{-1} \cdot \exp(-E_x/kT)$ to the center of the experimental points in Figure 5 and choosing the activation energy equal to its lowest possible value, the heat of the reaction E_x = 2.8 ev. When the dissociative recombination coefficient corresponding to the reverse reaction 1 is obtained by dividing the forward rate by the equilibrium constant, this procedure leads to a recombination rate with a $T_e^{-3/2}$ temperature dependence. Since this approach leads to the physically unrealistic situation where there is no electron energy loss on dissociative recombination, a different procedure will be adopted in the present analysis. A least-squares fit will be made of the functional form for the velocity-averaged associative ionization cross-section $\bar{Q} = AT^{-1/2} \exp(-B/T)$ corresponding to $\tau_{AI} \gg 10^{-14}$ sec to the experimental points given in Figure 5 of Lin and Teare.[4] Since the activation energy must be equal to or greater than the heat of the reaction, the restriction $B \geq 32,500$ will be added. A least-squares fit may be accomplished by minimizing the function

$$W = \sum_{k=1}^{N} [\bar{Q}(A, B, T_k) - Q_k]^2$$

where $\bar{Q}(A, B, T_k) = AT_k^{-1/2} \exp(-B/T_k)$ and Q_k (a constant)corresponds to the k^{th} experimental point at the temperature T_k (Figure 5). The minimization is achieved from the relations $\partial W/\partial A = 0$ and $\partial W/\partial B = 0$, which lead to the expressions:

$$\sum_{k=1}^{N} [\bar{Q}(A, B, T_k) - Q_k] \; Q(A, B, T_k) = 0$$

and

$$\sum_{k=1}^{N} [\bar{Q}(A, B, T_k) - Q_k] \; Q(A, B, T_k) = 0.$$

It is relatively easy to eliminate the parameter A from these two equations so that a single equation remains to determine the parameter B. Of course the result of the calculation must yield a value for B such that $B \geq 32,500$. Once the parameters A and B are determined from the least-squares fit, the associative ionization rate is determined from the formula

$$k_{f_1} = \sqrt{\frac{8kT}{\pi\mu}} \; \bar{Q}(A, B, T) = \sqrt{2.55 \frac{kT}{\mu}} \; AT^{-1/2} \exp(-\frac{B}{T}) \; .$$

The dissociative recombination rate is then obtained from the equilibrium constant

$$k_{r_1}(T_e) = \frac{\sqrt{2.55 \frac{k}{\mu}} \;\; A \; \exp(-\frac{B}{T})}{K_1^{EQ.}}$$

where

$$K_1^{EQ.} = (1.4 \times 10^{-8} T + 1.2 \times 10^{-12} T^2 + 1.4 \times 10^{-16} T^3) \exp(-32,500/T).$$

It should be mentioned that this procedure, while leading to a more realistic functional form for the dissociative recombination rate, suffers from the same weakness as the development of Lin and Teare.[4] Bates[62] has specifically pointed out that attempts to use the principle of detailed balancing to determine the cross section for associative ionization from measurements of the dissociative recombination cross section are not justified. This is because the systems in their initial states are usually in their ground state, whereas the final system is expected to be in an excited state. Nevertheless the principle of detailed balancing, though not strictly applicable, will be used in this report to obtain the dissociative recombination rate. Finally it will be assumed that the dissociative recombination coefficients of the reverse reactions 1, 2, and 3 will all be equal. The values thus obtained for these rate coefficients will be utilized until more reliable experimental information is available.

Energy loss of the electron gas due to three-body recombination:

$$2\pi m \sum \int_0^\infty \left[B_o^{\text{three-body recomb.}}(T_e) - B_o^{\text{three-body recomb}}(T)\right] v^4 dv$$

$$= N_e(T_e)\left(\frac{16}{3}\right) m\sqrt{2\pi}\, e^6 \left(\frac{1}{mkT_e}\right)^{3/2} \sum_k N_k(T_e)\cdot$$

$$\cdot\int_0^\infty \left[\Sigma G_j(v)\, \nu_j(v)\right] \frac{1}{v^2} \exp\left(\frac{-mv^2}{2kT_e}\right) dv - N_e(T)\left(\frac{16}{3}\right) m\sqrt{2\pi}\, e^6 (mkT)^{-3/2}$$

$$\sum N_k(T)\cdot \int_0^\infty \left[\Sigma G_j(v)\, \nu_j(v)\right] \frac{1}{v^2} \exp\left(\frac{-mv^2}{2kT}\right) dv$$

$$2\pi m \sum \int_0^\infty \left[B_o^{TBR}(T_e) - B_o^{TBR}(T)\right] v^4 dv$$

$$= \frac{2\pi}{k^2}\, e^6\, T_e^{-3/2}\, N_e(T_e) \sum N_k(T_e)\cdot$$

$$\cdot \sum \int_0^{T_e} \frac{1}{u^2} \int_0^u \frac{1}{v^2} \int_0^v W^{1/2}\, G_j(W)\, \nu_{j_{EFF}}(W)\, dw\,dv\,du$$

$$- \frac{2\pi}{k^2}\, e^6 T^{-3/2}\, N_e(T) \sum_k N_k(T) \sum_j \int_0^T \frac{1}{u^2} \int_0^u \frac{1}{v^2} \int_0^v W^{1/2}\, G_j(W)\, \nu_{j_{EFF}}(W)\cdot$$

Since only steady-state conditions are to be considered, the term $d/dt\ (3/2\ kT_e N_e)$ will equal zero and the energy balance equation becomes:

$$
\begin{aligned}
&\frac{N_e(T_e) e^2 \nu_{EFF} E^2}{m\omega^2} + 2kT_e N_e(T_e)\left(\frac{8kT_e}{\pi m}\right)^{1/2} \sum N_j \exp\left(\frac{-W_{ij}}{kT_e}\right)(\pi a_o^2) \\
&\quad + N_e(T_e)\left(\frac{8kT_e}{\pi m}\right)^{1/2} \sum N_j W_{ij} \exp\left(\frac{-W_{ij}}{kT_e}\right)(\pi a_o^2) \\
&\quad - 2kTN_e(T)\left(\frac{8kT}{\pi m}\right)^{1/2} \sum N_j \exp\left(\frac{-W_{ij}}{kT}\right)(\pi a_o^2) \\
&\quad - N_e(T)\left(\frac{8kT}{\pi m}\right)^{1/2} \sum N_j W_{ij} \exp\left(\frac{-W_{ij}}{kT}\right)(\pi a_o^2) \qquad (75) \\
&= \frac{3}{2}\left(\frac{2m}{M_{ion}}\right) N_e(T_e)\, k(T_e - T)(\nu_{ion})_{EFF} + \frac{3}{2}\sum_j G_j(T_e)\,[\nu_{EFF}(T_e)]_j\, kN_e(T_e)(T_e - T) \\
&\quad + \frac{3}{2} kT_e N_e(T_e) \sum N_i(T_e)\, k_i(T_e) + kT_e^2 N_e(T_e) \sum N_i(T_e) \frac{dk_i(T_e)}{dT_e} \\
&\quad - \frac{3}{2} kTN_e(T) \sum N_i(T)\, k_i(T) - kT^2 N_e(T) \sum N_i(T) \frac{dk_i(T)}{dT} \\
&\quad + \frac{2\pi}{k^2} e^6 T_e^{-3/2} N_e(T_e) \sum_k N_k(T_e) \sum_j \int_o^{T_e} \frac{1}{u^2} \int_o^u \frac{1}{v^2} \int_o^v W^{1/2} G_j(W) \nu_{j_{EFF}}(W)\, dw\,dv\,du \\
&\quad - \frac{2\pi}{k^2} e^6 T^{-3/2} N_e(T) \sum_k N_k(T) \sum_j \int_o^{T} \frac{1}{u^2} \int_o^u \frac{1}{v^2} \int_o^v W^{1/2} G_j(W) \nu_{j_{EFF}}(W)\, dw\,dv\,du.
\end{aligned}
$$

A summary of symbols used in the energy balance Eq. (75) follows: $N_e(T_e)$ is the electron density as a function of electron temperature given by Eq. (72). E^2 is the mean square value of the total field at a particular point in the plasma.

$$\nu_{EFF} = (\Sigma\, \nu_{en})_{EFF} + (\nu_{ion})_{EFF}$$

$$(\Sigma \nu_{en})_{EFF} = \frac{\sqrt{2}}{3\sqrt{\pi}} \left(\frac{m}{kT_e}\right)^{5/2} \int_0^\infty \sum N_j \sigma_j(v)\, v^5 \exp\left(\frac{-mv^2}{2kT_e}\right) dv$$

$$(\nu_{ion})_{EFF} = \frac{2}{3}\sqrt{\frac{8\pi}{m}}\; N_e(T_e)\, e^2 (kT_e)^{-3/2}\, \ell n(kT_e \ell_D / e^2)$$

$$\ell_D = \left[\frac{kTkT_e}{4\pi N_e e^2 (kT + kT_e)}\right]^{1/2} .$$

N_j refers to the concentration of the j^{th} neutral constituent. W_{ij} refers to the ionization potential of the j^{th} neutral constituent (Table 2)

$\pi a_o^2 = 0.876 \times 10^{-16} cm^2$

M is the averaged ionic mass = 24 amu

$G_j(T_e)$ is the fractional energy loss of an electron in collision with the j^{th} neutral constituent (Table 1)

$$[\nu_{EFF}(T_e)]_j = \frac{\sqrt{2}}{3\sqrt{\pi}} \left(\frac{m}{kT_e}\right)^{5/2} \int_0^\infty N_j \sigma_j(v) v^5 \exp\left(\frac{-mv^2}{2kT_e}\right) dv .$$

N_i refers to the concentration of the i^{th} molecular ionic constituent (NO^+, O_2^+, N_2^+) as a function of electron temperature.

$$(NO^+) = \frac{(N)(O)5\times10^{-11}T^{-1/2}\exp(-55,700/T) + (NO)\left(\frac{8kT_e}{\pi m}\right)^{1/2} \exp\left(\frac{-9.5}{kT_e}\right)(\pi a_o^2) N_e}{3\times10^{-3}\, T_e^{-3/2}\, N_e(T_e)}$$

$$(N_2^+) = \frac{\left\{(N)(N)[9\times10^{-11}T^{-1/2}(1+1.3\times10^{-4}T+3.3\times10^{-8}T^2-10^{-12}T^3)\cdot\exp(-67,300/T)] + (N_2)\left(\frac{8kT_e}{\pi m}\right)^{1/2}\exp\left(\frac{-15.7}{kT_e}\right)(\pi a_o^2) N_e\right\}}{3\times10^{-3}\, T_e^{-3/2}\, N_e(T_e)}$$

$$(O_2^+) = \frac{\left\{(O)(O)[3.2\times10^{-11}T^{-1/2}(1+7.5\times10^{-5}T+2.2\times10^{-8}T^2)\cdot\exp(-80,100/T)] + (O_2)\left(\frac{8kT_e}{\pi m}\right)^{1/2}\exp\left(\frac{-12.5}{kT_e}\right)(\pi a_o^2) N_e\right\}}{2\times10^{-3}\, T_e^{-3/2}\, N_e(T_e)}$$

N_k refers to the concentration of the k^{th} atomic ionic constituent (O^+, N^+) as a function of electron temperature.

$$(O^+) = \frac{(X)(O)4.62\times10^{-12}T^{1/2}\exp(-313,000/RT)+(O)\left(\frac{8kT_e}{\pi m}\right)^{1/2}\exp\left(\frac{-13.6}{kT_e}\right)(\pi a_o^2)\,N_e}{N_e(T_e)(X)T_e^{-3/2}\left(\frac{2\pi e^6}{k^3}\right)\sum\int_o^{T_e}\frac{1}{s^2}\int_o^{s}\frac{1}{u^2}\int_o^{u}\frac{1}{v^2}\int_o^{v}W^{1/2}G_j(W)\nu_{j_{EFF}}(W)dwdvduds}$$

$$(N^+) = \frac{(X)(N)4.86\times10^{-12}T^{1/2}\exp(-333,000/RT)+(N)\left(\frac{8kT_e}{\pi m}\right)^{1/2}\exp\left(\frac{-14.5}{kT_e}\right)(\pi a_o^2)\,N_e}{N_e(T_e)(X)T_e^{-3/2}\left(\frac{2\pi e^6}{k^3}\right)\sum_j\int_o^{T_e}\frac{1}{s^2}\int_o^{s}\frac{1}{u^2}\int_o^{u}\frac{1}{v^2}\int_o^{v}W^{1/2}G_j(W)\nu_{j_{EFF}}(W)dwdvduds.}$$

For a given value of the mean-square electromagnetic field (E^2), the electron temperature at a particular point in the plasma may be determined by solving the energy balance equation [Eq. (75)] for (T_e). All of the parameters which appear in the energy balance equation are capable of being expressed as functions of electron temperature in analytic form, except the terms $(\nu_{en})_{EFF}$, $G_j(T_e)$, and the energy loss of the electron gas associated with three-body recombination. However these terms are known or may be evaluated in graphical form as functions of electron temperature. Then, it is always possible to construct a relatively simple analytic expression to fit the known graphical form of these functions. This permits the energy balance equation to be solved for the electron temperature once the intensity of the electromagnetic field is fixed. When the electron temperature has been determined, the electron density may be computed from Eq. (72). As soon as the electron temperature and density have been determined, the dielectric constant (K) for this particular point in the plasma may be found by using Eq. (67). Finally, on the basis of the model of a plane-layered plasma medium, the electromagnetic field distribution in the nonlinear plasma slab may be computed step-by-step by utilizing the energy balance Eq. (75), the expression (72) for the electron density, and expression (67) for the dielectric constant in conjunction with Maxwell's equations expressed in the form of difference equations.

2.4 Heat Transport Through the Electron Gas Due to Conduction and Particle Diffusion

The various energy loss processes of the electron gas due to heat conduction and particle diffusion have been carefully investigated by Anderson and Goldstein.[9, 41] These investigations were devoted mainly to the study of cross modulation

(Luxembourg) effects in the afterglow of pulsed gaseous discharge plasmas. Electron-neutral collision frequency changes are produced by heating the plasma in the afterglow through the application of a heating microwave pulse. These electron collision frequency changes are observed by the variations in attenuation they produce on a second or 'wanted' probing electromagnetic wave. Anderson and Goldstein[9] properly note that an energy balance equation for a particular region of the electron gas must include terms which account for energy loss due to heat transport through (1) electron-electron collisions (Spitzer and Harm[10] coefficient), (2) electron-neutral collisions, and (3) particle diffusion. Hence, in addition to the enumerated electron gas energy gain and loss terms which account for energy exchange between the electromagnetic field, the electron gas, and the neutral and ion gas [Eq. (75)], energy transport from one part of the electron gas to another due to gradients in electron temperature and electron density must also be examined.

Anderson and Goldstein[9] have given an explicit expression for the heat transfer coefficient due to electron-neutral and electron-ion collisions which was first derived by Gould:[47]

$$K_{eN} = \frac{64\, K^2 T_e N_e}{9\pi m\, \nu_{EFF}} \tag{76}$$

where ν_{EFF} is given by Eq. (2) and $\nu(v) = \Sigma\nu_{eN} + \nu_{ion}$. The heat conductivity coefficient due to electron-electron collisions (K_{ee}) is larger than K_{eN} because the interelectron collision frequency is high and the Coulomb forces have a long range. Spitzer and Harm[10] have derived an expression for the heat conduction coefficient for the case of relatively small temperature gradients in a plasma:

$$K_{ee} = \frac{20\, m^2 k c^5}{3\, e^4 \ln (qc^2)} \left(\frac{2}{3\pi}\right)^{3/2} \delta T \tag{77}$$

where $c = (3k\, T_e/m)^{1/2}$, $\ln(q\, c^2) \approx 6$ when $N_e = 10^{12}/\text{cm}^3$

$\delta_T = 0.225$ $\qquad T_e = 3000°K$.

Finally, heat transport may occur due to the existence of electron density gradients in the plasma. This term assumes the form:

$$\frac{64\, k^2\, T_e}{9\pi m\, \nu_{EFF}}\, \nabla N_e \cdot \nabla T_e \quad . \tag{78}$$

It should be remembered that there will be considerable heat transport at equilibrium across the sharp density and temperature gradients which exist at the shock front and at the boundary layer. This heat flow will proceed, of course, even in the absence of an electromagnetic wave. This heat flow is of no consequence in so far as the determination of the steady state electron temperature in the presence of the electromagnetic field is concerned, because it is only those density and temperature gradients established by the electromagnetic field over and above the gradients which had originally existed at equilibrium which will be responsible for loss at a particular point in the plasma of the heat deposited by the electromagnetic field. These considerations lead to heat transport terms which may be written:

(1) Flow of heat deposited by electromagnetic wave due to electron-neutral and electron-ion collisions

$$\frac{64k^2T_eN_e(T_e)}{9\pi m\,\nu_{EFF}(T_e)}\,\nabla^2T_e - \frac{64k^2TN_e(T)}{9\pi m\,\nu_{EFF}(T)}\,\nabla^2T$$

(2) Flow of heat deposited by electromagnetic wave due to electron-electron collisions

$$K_{ee}(T_e)\,\nabla^2T_e - K_{ee}(T)\,\nabla^2T$$

(3) Flow of heat deposited by electromagnetic wave due to density gradients

$$\frac{64k^2T_e}{9\pi m\,\nu_{EFF}(T_e)}\,\vec{\nabla N}_e\cdot\vec{\nabla T}_e - \frac{64k^2T}{9\pi m\,\nu_{EFF}(T)}\,\vec{\nabla N}_e\cdot\vec{\nabla T}\ .$$

The nonlinear reflection and transmission coefficients for a plane wave incident at an arbitrary angle upon a plane-layered inhomogeneous plasma slab having the characteristics of the re-entry sheath may be computed by determining the electromagnetic field distribution in the slab by integrating Maxwell's equations by a step-by-step procedure. The step-by-step numerical integration of the field equations entails the solution of an energy balance equation at each step. The most general form for the energy balance equation at an arbitrary point in the plasma slab includes the following terms:

$$\left\{\begin{matrix}\text{Energy deposited by}\\ \text{e. m. field in plasma}\\ \text{by ohmic heating}\end{matrix}\right\} + \left\{\begin{matrix}\text{Energy gain of electron}\\ \text{gas due to new particles}\\ \text{created by electron-}\\ \text{neutral impact}\end{matrix}\right\} =$$

$$\left\{\begin{array}{l}\text{Energy loss of electron gas}\\\text{due to dissociative re-}\\\text{combination}\end{array}\right\} + \left\{\begin{array}{l}\text{Energy loss of electron}\\\text{gas due to three-body}\\\text{recombination}\end{array}\right\} +$$

$$\left\{\begin{array}{l}\text{Energy loss due to elastic}\\\text{electron-neutral and electron-}\\\text{ion collisions and inelastic}\\\text{electron-neutral collisions}\\\text{such as collisions which excite}\\\text{rotational, vibrational, and}\\\text{optical levels and ionizing}\\\text{collisions}\end{array}\right\} + \left\{\begin{array}{l}\text{Energy transport due}\\\text{to electron-neutral}\\\text{and electron-ion}\\\text{collisions}\end{array}\right\} +$$

$$\left\{\begin{array}{l}\text{Energy transport due}\\\text{to electron-electron}\\\text{collisions}\end{array}\right\} + \left\{\begin{array}{l}\text{Energy transport due}\\\text{to particle diffusion}\end{array}\right\} .$$

If the last three energy transport terms in the above equation are negligible compared to the terms that represent heat flow between the electron gas and neutral gas, then the fact that the neutral gas constitutes an infinite heat reservoir implies that there is no coupling of energy between the different layers of the plane-layered medium. In spite of the fact that the heat transport coefficients may be large (especially the heat conductivity coefficient due to electron-electron collisions), if the electron temperature gradients established by the electromagnetic field are not drastically different from the gradients which existed in the absence of the field, then the steady-state electron temperature within an arbitrary layer of the plane-layered medium will not be a function of the temperature at adjacent points. Rather, the electron temperature, determined from the energy balance Eq. (75), will depend solely upon the rate of heat flow into the electron gas due to ohmic heating and the heat flow out of the electron gas to the neutral and ion gas through the various elastic and inelastic collision processes.

It should be emphasized that the effects of heat flow from the electron gas through the boundary layer to the vehicle's surface have been entirely neglected in the present analysis. Since most of the streamtube approximations (Lin and Teare,[48] Rotman and Meltz[11]) neglect diffusion from streamtube to streamtube (constant enthalpy along a streamtube) in the process of calculating electron density for equilibrium and nonequilibrium flow regimes, the standard techniques for computing electron density profiles about hypersonic re-entry vehicles contain no inherent mechanism which can account for heat transport across the ionized layer. Bond[34] has investigated the approach to equilibrium ionization in argon shocks. His analysis leads to the prediction of charge separation across the shock front due to the difference in mobilities of positive ions and electrons. The electron density gradient was responsible for the electron diffusion. This effect will also be neglected in this report. In addition, for the well-defined shock fronts characteristic of blunt-nosed re-entry vehicles at 200,000 ft traveling at 18,000 ft/sec, it will be assumed that the gradients of electron temperature and electron density in the boundary layer and at the shock front will be so steep compared to a wavelength (3 cm for X-band) that they may be represented as plane interfaces. By examining the heat transport coefficients represented by Eqs. (76), (77), (78), together with the electron temperature and electron density gradients usually encountered behind the shock of a blunt-nosed re-entry vehicle at 200,000 ft traveling at 18,000 ft/sec (see Figures 18, 19, and 20 in the report by Rotman and Meltz[11]), it is easily seen that the largest heat transport term is given by $K_{ee}(T_e)\, \nabla^2 T_e - K_{ee}(T)\nabla^2 T$, representing heat conductivity due to electron-electron collisions. Assuming that the electron temperature and electron density gradients established by the electromagnetic field are not greater than ten times the corresponding gradients that prevailed in the absence of the field, it may be demonstrated that the largest heat transport term corresponding to the greatest change in temperature gradient is at least two orders of magnitude less than the term representing heat loss of the electron gas through elastic and inelastic collisions.

The orders of magnitude of these quantities are listed below:

$$N_e = 10^{18} \text{ elec/m}^3$$

$$T_e \approx 10^4 \text{ °K}$$

$$T = 5000\text{°K}$$

$$\nabla^2 T_e \leq 10^6 \text{ °K/m}^2$$

$$K_{ee}(T_e) \approx 10^{-2} \text{ joules/m}^2\text{/sec/°K/m}$$

$$G_{EFF} \leq 10^{-2}$$

$$\nu_{EFF} \approx 10^9 \text{ coll/sec}$$

$$K_{ee}(T_e)\, \nabla^2 T_e - K_{ee}(T)\nabla^2 T \approx 10^4 \text{ joules/sec - m}^3$$

$$3/2\, G_{EFF}\, \nu_{EFF}\, N_e\, k(T_e - T) \approx 10^6 \text{ joules/sec - m}^3 \quad .$$

The effects of heat conduction may be taken into account as a second order correction to the electron density and temperature profile across the layer of ionized gas. The procedure involves an iterative technique to be described in Section 5.

2.5 Diffusion Effects

It should be noted that no mass motion of the plasma is assumed in the analysis. This is essentially the result of the fact that the plane wave, which is incident upon the plasma slab, has an infinite extent. Hence there is no flow of plasma out of a finite region over which the ohmic heating occurs. The electrons which are created on the spot by the induced changes in the reaction rates are assumed not to diffuse across the layer of ionized gas. This is a very good assumption since, within the layer of ionized gas, strong electrostatic restoring forces will severely inhibit the flow of electrons out of the region in which they were created. Diffusion is characterized by strong coupling between electron and ion motions (ambipolar), since the Debye length ℓ_D is much less than the shock layer thickness. The positive ion motion is completely determined by the structure of the flow field of the neutral gas. The Debye length for electron densities of 10^{12} elec/cm^3 and electron temperatures of 10,000°K is about 10^{-3}cm, or less than 10^{-4} times the shock thickness. The conditions for ambipolar diffusion are satisfied, which implies that there will be a space charge sheath of electrons at the boundaries of the plasma slab (shock front and boundary layer). It will be assumed that this electron layer buildup at the boundaries has a negligible effect on the microwave transmission characteristics of the re-entry plasma sheath.

3. KINETICS OF HIGH TEMPERATURE AIR IN THE PRESENCE OF AN ELECTROMAGNETIC FIELD

There have been some efforts devoted to the calculation of enhanced ionization of the re-entry plasma sheath under the influence of intense microwave radiation. King and Gray,[81] King,[82] and Sodha[83] have computed the enhanced degree of ionization of the plasma sheath under the influence of high-power electromagnetic radiation on the basis of the Saha equation. The Saha equation relates the electron density in a plasma at the $(j + 1)^{th}$ stage of ionization to the temperature, partition function, and ionization potential of the j^{th} stage of ionization. The usual form of Saha's equation (Bachynski et al.[38]) applies only to the case of thermodynamic equilibrium, and is derived under the assumption that the distribution function for particle energies is Maxwellian and that the electron temperature equals the gas temperature. Dewan[84] has generalized the Saha equation to include any steady-state equilibrium situation in a plasma. Dewan demonstrates that the steady-state equilibrium condition implies that the rate of collision and radiation ionization processes for a given stage of ionization must equal the rate of collision and radiative recombination processes for that stage. This leads to an expression relating the ion densities to the radiation and particle energy distribution functions. Dewan[84] has specifically pointed out that attempts to apply the usual form of the Saha equation to physical situations where thermodynamic equilibrium does not pertain can lead to totally erroneous results. Dewan's approach, which ignores collisions between heavy particles, may be applied to a multicomponent plasma when charge exchange is negligible. For the situation considered in the present analysis, where the plasma is transparent and ionization due to recombination radiation is negligible, Dewan's approach reduces to the steady-state particle conservation equation obtained by taking the zero-order moment of the Boltzmann equation. An accurate picture of microwave-enhanced ionization of the plasma sheath may be obtained only by studying the numerous particle-particle and field particle interactions. Such a detailed study will then reveal the relative importance of the competing ionization processes.

The complete description of the kinetics of the appropriate collisional processes and the kinetics of chemical reactions occurring in a multicomponent plasma, such as shock-heated air, is an exceedingly difficult problem. The complexity is due in part to the simultaneous occurrence of translational, rotational, and vibrational relaxation behind the shock front, vibrational and radiative excitations, dissociation and ionization. When an electromagnetic field of high intensity is impressed upon an ionized flow field, the rates of reaction of many of these processes will be changed. Nevertheless the problem is not intractable, primarily because many of these processes can be assumed to occur independently of the others to a high degree of accuracy. Translational and rotational equilibrium of N_2 and O_2 are achieved within a distance of about one upstream mean free path behind a normal shock in air (ℓ_1 = 0.05 cm at an altitude of about 200,000 feet). The computations of Lin and Teare[4] demonstrate that translational/vibrational equilibrium is established at a distance of about 100 upstream mean free paths. The processes of vibrational relaxation and dissociation cannot be uncoupled, because molecules in excited vibrational states dissociate more readily than unexcited ones.[49] Lin and Teare's[4] investigations of the various competing ionization processes behind the shock front show that the relative importance of these processes follows the order:

(1) Atom-atom impact
(2) Photoionization
(3) Electron impact on neutrals
(4) Molecule-molecule and atom-molecule impact.

This order is preserved for most of the region behind an air shock at 0.02 mm Hg (200,000 ft) at a velocity of about 20,000 ft/sec, although as Lin and Teare point out, the rapid initial rates of change in temperature and chemical composition behind the shock front can cause a shift in the relative order of importance of the competing ionization processes. Since atom-atom impact is the dominant electron production mechanism, the rate of production of electrons behind the shock depends upon the translational/vibrational equilibrium and the degree of dissociation.

Lin and Teare consider the following processes to be the chief chemical reactions for high temperature air:

$$
\begin{aligned}
&(1) \quad O_2 + X + 5.1\ \text{ev} \rightarrow O + O + X \\
&(2) \quad N_2 + X + 9.8\ \text{ev} \rightarrow N + N + X \\
&(3) \quad NO + X + 6.5\ \text{ev} \rightarrow N + O + X \\
&(4) \quad O + N_2 + 3.3\ \text{ev} \rightarrow NO + N \\
&(5) \quad N + O_2 - 1.4\ \text{ev} \rightarrow NO + O \\
&(6) \quad N_2 + O_2 + 1.9\ \text{ev} \rightarrow NO + NO\ .
\end{aligned}
\tag{79}
$$

Since the degree of ionization for shocks up to 30, 000 ft/sec is always small (less than 1 per cent), the electron production and recombination processes will not affect any of the neutral reactions (translational-vibrational equilibration and dissociation). Hence Lin and Teare were able to compute the vibrational and translational temperatures and the particle densities as a function of distance behind a one-dimensional shock by solving the chemical rate Eq. (79) simultaneously with the vibrational energy relaxation equations for O_2 and N_2 together with the mass, momentum, and energy conservation equations. A knowledge of the reaction rate coefficients for the numerous electron production processes enable them to compute the electron and ion densities as a function of distance behind the shock front.

If a high-power electromagnetic wave is impressed upon the ionized flow field, the translational-vibrational equilibration process and the dissociation reactions (79) will not be influenced. This is due essentially to what Lin and Teare term 'the one-way coupling' between the dissociation process and the ionization processes. Since the fractional energy loss of an electron per collision with a neutral (G) is always less than 10^{-2} even up to electron temperatures of 20, 000°K and the degree of ionization is 0. 1 per cent or less, the electron gas does not affect the energy balance or the reaction rates of the atomic and molecular processes of the neutral constituents. Only the ionization and electron recombination process will be affected by the presence of the high-power electromagnetic field. For example an intense microwave field will alter the relative importance of the competing ionization processes by increasing the electron temperature so that ionization by electron impact on neutrals will become more important than photoionization, and even as important or more important than atom-atom collisions. This would be most pronounced at least ten upstream mean free paths behind the shock front where the electron density is built up to an appreciable level (see Figure 13, Lin and Teare[4]). One important point that should be emphasized is that, in the present analysis, a basic consideration rests upon the assumption that the electron gas is in thermal equilibrium with the neutral gas in the absence of an electromagnetic field. This assumption is almost always made in computations of the electron density profiles behind a shock front (Lin and Teare,[4] Rotman and Meltz,[11] and Bortner[50]). Petschek and Byron[51] and Bond[52] have investigated the onset of ionization and the approach to equilibrium in argon shocks. Here the dominant electron production mechanism appears to be electron-neutral impact. It was found that during the approach to equilibrium, the electron temperature is less than the gas temperature. The distance over which this equilibration takes place behind a shock front in air may be estimated in the following manner. The fraction of the average energy transferred between an electron and neutral (G) per collision is of the order 10^{-2}. Hence, in about 100 electron-neutral mean free paths the electrons should be in equilibrium with the neutral particles. For a 5000°K shock corresponding to a blunt-nosed re-entry vehicle at 200,000 ft traveling at 20,000 ft/sec, this distance would extend approximately 1 cm behind the shock front. Since this distance is short compared to a wavelength (3 cm at X-band), the representation of the gradients and discontinuities at the shock front by plane interfaces is a good approximation. This assumption is better justified at 100,000 feet.

The various ionization processes which may occur behind a shock in air include:

(1) Ionization by atom-atom, atom-molecule, and molecule-molecule collisions

(2) Photoionization

(3) Electron impact on neutrals and ions.

Also, charge exchange can influence the total electron density because electron-ion recombination is specie dependent. Electron attachment to neutrals is another possible loss mechanism, because a heavy negative ion has little influence on the microwave conductivity of the plasma.

3.1 Neutral - Neutral Impact

Perhaps the most surprising result of Lin and Teare's study of the ionization processes behind an air shock is the fact that atom-atom impact is the dominant electron production mechanism for shock velocities up to about 30,000 ft/second. This is an unexpected result, for studies of ionization processes in argon shocks (Petschek and Byron[51] and Bond[34]) indicated that electron-atom impact was the dominant mechanism. The minimum activation energy for producing ionization by atom-atom impact corresponds to the reaction:

$$N + O + 2.8\ ev \rightarrow NO^+ + e.$$

The concentration of atomic species will not reach appreciable levels until the dissociation of O_2 and N_2 is fairly complete. For example, for a shock corresponding to an altitude of 200,000 ft at a velocity of 20,000 ft/sec, dissociation of O_2 and N_2 have attained 75 per cent and 50 per cent of their respective final equilibrium level at a distance of ten mean free paths behind the shock front. At this distance the translational temperature of the gas particles has dropped to about 1 ev. In spite of activation energies which are considerably greater than 1 ev, ionization by atom-atom impact is an important process because of the relatively large velocity-dependent cross sections. The cross sections are large because of the crossing of the potential energy curves representing the interaction potential of the atomic system before and after the collision. This cross section may be determined (see Bates and Massey[46]) by considering the fact that there is a relatively large probability for a transition to occur between the initial and final states of the atomic system at the point where their respective potential energy curves cross. This probability is related to the eigenfunctions of the quasi-molecule formed by the colliding systems, the Hamiltonian of the colliding system, and the internuclear distance at which the curve crossing occurs (Landau-Zener formula[63, 64]).

Lin and Teare[4] have considered the following ionization processes due to neutral-neutral impact:

$$
\begin{aligned}
&(1)\quad N + O + 2.8\ \text{ev} \rightarrow NO^+ + e \\
&(2)\quad N + N + 5.8\ \text{ev} \rightarrow N_2^+ + e \\
&(3)\quad O + O + 6.9\ \text{ev} \rightarrow O_2^+ + e \\
&(4)\quad X + O + 13.6\ \text{ev} \rightarrow X + O^+ + e \\
&(5)\quad X + N + 14.6\ \text{ev} \rightarrow X + N^+ + e \\
&(6)\quad N + O_2 + 6.5\ \text{ev} \rightarrow NO_2^+ + e \\
&(7)\quad O + NO + 7.9\ \text{ev} \rightarrow NO_2^+ + e \\
&(8)\quad N + NO + 7.9\ \text{ev} \rightarrow N_2O^+ + e \\
&(9)\quad X + NO + 9.3\ \text{ev} \rightarrow X + NO^+ + e \\
&(10)\quad O + N_2 + 11.2\ \text{ev} \rightarrow N_2O^+ + e \\
&(11)\quad O + O_2 + 11.7\ \text{ev} \rightarrow O_3^+ + e \\
&(12)\quad X + O_2 + 12.1\ \text{ev} \rightarrow X + O_2^+ + e \\
&(13)\quad X + N_2 + 15.6\ \text{ev} \rightarrow X + N_2^+ + e \\
&(14)\quad N_2 + O_2 + 11.2\ \text{ev} \rightarrow NO + NO^+ + e\ .
\end{aligned}
\tag{80}
$$

By examining the rates reported in the literature, taken in conjunction with theoretical considerations on the potential curve crossing of atomic systems and the equilibrium relationship, Lin and Teare have selected values for most of the forward and backward rates of the neutral-neutral impact ionization reactions (80). By and large, the values chosen for these rates appear to be reasonable, except for reactions (1), (2), and (3) of Eq. (80). The temperature dependence of $T_e^{-3/2}$ selected for the dissociative recombination rates corresponding to the backward reactions (1), (2), and (3) requires that the velocity dependent cross section assume a physically unrealistic form. Hence, in the present analysis, the rate for the forward reaction (1) is determined by making a least-squares fit of the functional form $AT^{-1/2} \exp(-B/T)$ to the experimental data of Lin[45] (see Figure 5, Lin and Teare[4]). This procedure is described in detail in Section 2. 3. 2. The backward rate for reaction (1) is obtained from the equilibrium relationship The backward rates for the dissociative reactions 1, 2, and 3 are assumed equal. Then, the forward rate constants for the associative ionization reactions 2 and 3 were obtained through the equilibrium relationship.

In the present investigation, atom-molecule and molecule-molecule impact ionization processes are neglected (except reactions 4 and 5). This is justified on the basis of Lin and Teare's estimations of the specific ionization rates for each of the various ionizing processes. The specific ionization rate is defined as the absolute value of the time rate of change of the normalized electron density multiplied by the quantity (ℓ_1/U_s), where

ℓ_1 = upstream mean free path = 0. 05 cm at an altitude of 200, 000 ft

U_s = shock velocity.

Figure 8b from the report of Lin and Teare[4] indicates that for a shock at 200, 000 ft traveling at 20, 000 ft/sec, atom-molecule and molecule-molecule impact ionization processes are quite negligible compared with ionization by atom-atom impact for most of the region behind the shock front. Consequently, in this report, only the ionization processes due to neutral-neutral impact corresponding to reactions (1), (2), (3), (4), and (5) of Eq. (80) will be considered. The reaction rate coefficients for the forward reactions (4) and (5) are taken from the estimates made by means of the collision theory used by Bortner.[39] The three-body recombination rates for the reverse reactions (4) and (5) are based upon the Thomson theory[36] generalized to the case of a multicomponent plasma in which the electron temperature is not equal to the gas temperature. In conclusion it might be mentioned that in addition to the forward reactions (4) and (5), both Bortner[39] and Atallah[40] have estimated that the following reactions are capable of producing O^+ and N^+ at quite a rapid rate:

$$(1) \quad N + NO \rightarrow O^+ + N_2 + e$$
$$(2) \quad O + NO \rightarrow N^+ + O_2 + e \qquad . \qquad (81)$$

However, Lin and Teare have neglected these reactions, and since reliable rate constants for these reactions are not available, they will be neglected in this report.

3.2 Photoionization

The rate at which electrons are produced per unit volume for a particular fluid element behind a shock due to photoionization depends upon the number densities of the atomic and molecular species, their photoionization cross sections, and the photon number density. The far ultraviolet appears to be the spectral region which is most effective in producing photoionization of N_2, O_2, and NO. It is only through a consideration of excitation, emission, and radiative transport phenomena that the spectral photon density for a particular fluid element may be determined. This difficult problem has not been solved by Lin and Teare.[4] Instead a somewhat crude estimate has been made of the electron production rate due to photoionization. On the basis of measurements made by Camm et al.,[53] Hammerling[54] has deduced that the major contribution to the far ultraviolet radiation emanating from an air shock may be attributed to the $b' \, {}^1\Sigma_u^+ \rightarrow X \, {}^1\Sigma_g^+$ transition of N_2.

This fact has enabled Lin and Teare[4] to obtain an expression for the total intensity of ultraviolet radiation per unit area emanating from the shock. Photoionization cross sections of N_2, O_2, and NO have been taken from the data of Weissler et al.,[55] and Wainfan et al.[56] The energy of the photons corresponding to the $b' \, {}^1\Sigma_u^+ \rightarrow X \, {}^1\Sigma_g^+$ transition of N_2 (12.9 ev) is such that this ultraviolet radiation is capable of ionizing only the molecules O_2 and NO. These facts lead to the following approximate expression for electron production due to photoionization (Lin and Teare[4]).

$$\frac{dN_e}{dt} \approx \frac{\pi a_o^2 \, [(O_2) + (NO)] \, \overline{(N_2)} \; L \exp(-E^* / kT^*)}{8 \, (\tau_r + \tau_e)} \qquad (82)$$

where

τ_r = radiative life time of upper electronic state of N_2

= 4×10^{-9} sec

τ_e = collisional de-excitation time

E^* = energy difference between two electronic states of N_2

= 12.9 ev

L = effective length of a particular fluid element

T^* = an effective temperature that determines the population of the electronic states of N_2.

By using Eq. (82), Lin and Teare[4] were able to illustrate the effect of photoionization on the electron density profile behind an air shock corresponding to a velocity of about 20,000 ft/sec at an altitude of 200,000 feet. Figure 12 of the report of Lin and Teare[4] demonstrates that for distances behind the shock front greater than one upstream mean free path, photoionization produces a negligible contribution to the total electron density.

3.3 Ionization by Electron Impact

Since the Maxwellian distribution function has a long tail at high electron velocities, ionization produced by electron-neutral impact will become comparable with associative ionization by atom-atom impact when the electron energy becomes equal to or greater than about one-sixth the ionization potential of a neutral constituent (NO has the lowest ionization potential: 9.25 ev). Equation (68b) describes the rate at which electrons are created by electron-neutral impact. The right-hand side of Eq. (68b) has been obtained from the integral expression for impact ionization by expanding the ionization cross section in powers of $(\epsilon - W_i)$, and neglecting second-order terms in (kT/W_i). Here, $\epsilon = \frac{1}{2} mv^2$ is the electron energy, W_i = ionization potential of neutral constituent, and kT has been assumed to be much less than the ionization potential ($kT \ll W_i$).

$$\frac{dN_e}{dt} = 4\pi \sum N_j \int_o^\infty f_o \, \sigma_{ion_j} \, v^3 dv$$

$$= N_e \left(\frac{1}{kT_e}\right)\left(\frac{8}{\pi m kT_e}\right)^{1/2} \sum N_j \int_{W_{ij}}^\infty \exp\left(\frac{-\epsilon}{kT}\right) \sigma_{ion_j}(\epsilon)\epsilon d\epsilon$$

$$= N_e \left(\frac{1}{kT_e}\right)\left(\frac{8}{\pi m kT_e}\right)^{1/2} \sum N_j \int_{W_{ij}}^\infty \exp\left(\frac{-\epsilon}{kT}\right) \cdot$$

$$[\sigma_{ion_j}(W_{ij}) + (\epsilon - W_{ij})\, \sigma'_{ion_j}(W_{ij}) + \dots]\, \epsilon d\epsilon$$

$$= N_e \left(\frac{8kT_e}{\pi m}\right)^{1/2} \sum N_j \exp\left(\frac{-W_{ij}}{kT_e}\right) W_{ij} \quad \sigma'_{ion_j} (W_{ij})$$

where $\sigma_{ion\ j} (W_{ij}) = 0$ and $W_{ij}\ \sigma'_{ion\ j} (W_{ij}) = \pi a_o^2$. This result was first obtained by Lin.[57] The neutral constituents which will be considered in this analysis include: N_2, O_2, N, O, and NO. The right-hand side of Eq. (68b) should give a fairly accurate estimation of the electron production rate due to electron-neutral impact for electron energies up to about 2 ev (20,000°K).

It might be mentioned that the mechanism of electron production by electron impact on neutrals in excited metastable electronic states has been neglected.

3.4 Effects of Charge Exchange

The effects of charge exchange on the total electron density behind an air shock will be manifested indirectly. The presence of an electromagnetic field in the ionized flow field does not influence the rates of reaction for the charge transfer processes, for the gas temperature is not changed. Charge exchange involves an alteration of the relative concentrations of the various ionic species. The electron density will be affected if the electron-ion recombination rates are different for different species. The somewhat meagre experimental data on dissociative recombination rates seem to indicate that the recombination coefficient is the same for the species NO^+, N_2^+, and O_2^+. This implies that the electron density would be affected only by charge exchange between the molecular and atomic species:

$$X^+ + YZ \rightarrow YZ^+ + X \quad . \tag{83}$$

This type of reaction (83) is termed asymmetrical nonresonant charge transfer, for which the cross section at 2 ev for an Li^+ – Argon system is about $10^{-20}cm^2$ (Fogel et al.[58]). Most nonresonant charge transfer processes are quite inefficient at thermal energies. However there is another type of reaction, termed asymmetrical resonant charge transfer, which may have a larger cross section at thermal energies. An example of asymmetric resonant charge transfer has been suggested by Omholt:[59]

$$O^+(^2D) + N_2\ (X\ ^1\Sigma^+_{g_1}\ v = 0) \rightarrow O(^3P) + N_2^+\ (A^2\pi_u,\ v = 1) \quad . \tag{84}$$

According to Bates and Lynn,[60] it is improbable that this reaction has a large cross section at thermal velocities.

But, as Bates[61] has indicated, even though charge transfer is rather inefficient at thermal energies, ion-atom interchange may be quite rapid:

$$X^{+} + XZ \rightarrow XY^{+} + Z \quad . \tag{85}$$

It is possible that the rate coefficient for ion-atom interchange may be as large as $10^{-9} cm^{3}/sec$ at thermal energies, provided the activation energy is small.

Hence, it would seem that charge exchange could influence the steady-state electron density if the following typical chain of reactions occurred.

$$\begin{aligned} &(a) \qquad O^{+} + N_2 \rightarrow NO^{+} + N \\ &(b) \qquad NO^{+} + e \rightarrow N + O \quad . \end{aligned} \tag{86}$$

The ion-atom interchange reaction (86a) could be fairly rapid, and the dissociative recombination reaction (86b) proceeds much more rapidly than the three-body recombination of electrons with O^{+}. Only the rate of the ion-atom interchange reaction

$$O^{+} + O_2 \rightarrow O_2^{+} + O \tag{87}$$

has been repo.*ed in the literature. Dickinson and Sayers[65] have measured a rate of $10^{-11} cm^{3}/sec$ at 300°K, which implies a cross section of about $2 \times 10^{-18} cm^{2}$.

Lin and Teare[4] have listed the following charge exchange reactions:

(1) $N_2^{+} + N \rightarrow N_2 + N^{+} + 1.0$ ev

(2) $N_2^{+} + O \rightarrow N_2 + O^{+} + 2.0$ ev

(3) $N_2^{+} + O_2 \rightarrow N_2 + O_2^{+} + 3.5$ ev

(4) $N_2^{+} + NO \rightarrow N_2 + NO^{+} + 6.3$ ev

(5) $N^{+} + O \rightarrow N + O^{+} + 0.9$ ev

(6) $N^{+} + O_2 \rightarrow N + O_2^{+} + 2.5$ ev

(7) $N^{+} + NO \rightarrow N + NO^{+} + 5.3$ ev

(8) $O^{+} + O_2 \rightarrow O + O_2^{+} + 1.6$ ev

$$\text{(9)} \qquad O^+ + NO \rightarrow O + NO^+ + 4.4 \text{ ev}$$

$$\text{(10)} \qquad O_2^+ + NO \rightarrow O_2 + NO^+ + 2.8 \text{ ev} \quad .$$

These reactions are all of the asymmetric nonresonant type, except the reverse reaction (2), which may be of the asymmetric resonant type. Lin and Teare have taken the cross section for all the exothermic reactions to equal $3 \times 10^{-16} cm^2$. This is a couple of orders of magnitude larger than the asymmetric nonresonant charge transfer cross sections measured by Fogel et al.[58] in Li^+ – Argon systems at 2 ev. However, Lin and Teare[4] have investigated the effects of varying the exothermic charge exchange cross section on the electron density and positive ion density profiles behind an air shock at about 200, 000 ft, corresponding to a velocity of 20, 000 ft/second. Figure 11 of the report by Lin and Teare[4] demonstrates that changes in the charge transfer cross sections of factors of 100 in either direction produce changes of about 10 per cent or less in the density profiles of electrons and all positive ions except O_2^+. For distances between 1 and 100 upstream mean free paths behind the shock front, the ionic specie O_2^+ exhibits about a 400 per cent change in density when the charge exchange cross sections are varied by a factor of 100.

Lin and Teare[4] have listed the following ion-atom interchange reactions and charge rearrangement reactions:

$$
\begin{array}{ll}
\text{(1)} & N_2^+ + O \rightarrow NO + N^+ - 2.2 \text{ ev} \\
\text{(2)} & N_2^+ + O \rightarrow NO^+ + N + 3.0 \text{ ev} \\
\text{(3)} & N_2 + O^+ \rightarrow NO + N^+ - 4.2 \text{ ev} \\
\text{(4)} & N_2 + O^+ \rightarrow NO^+ + N + 1.1 \text{ ev} \\
\text{(5)} & N_2^+ + O_2 \rightarrow NO + NO^+ + 4.5 \text{ ev} \\
\text{(6)} & N_2 + O_2^+ \rightarrow NO + NO^+ + 0.9 \text{ ev} \\
\text{(7)} & O_2^+ + N \rightarrow NO + O^+ - 0.2 \text{ ev} \\
\text{(8)} & O_2^+ + N \rightarrow NO^+ + O + 4.2 \text{ ev} \\
\text{(9)} & O_2 + N^+ \rightarrow NO + O^+ + 2.3 \text{ ev} \\
\text{(10)} & O_2 + N^+ \rightarrow NO^+ + O + 6.7 \text{ ev} \quad .
\end{array}
\tag{89}
$$

Although no experimental data on the rates of reactions (89) has been reported in the literature, the ion-atom interchange reaction (87) has a measured cross section of about $10^{-18}cm^2$ at 0.03 ev. This result may be interpreted to imply that the ion-atom interchange processes [Eq. (89)] have a cross section which is one or perhaps two orders of magnitude greater than the cross section for asymmetric non-resonant charge transfer [Eq. (88)]. Lin and Teare[4] have taken the cross section for the exothermic charge rearrangement reactions (89) to be equal to about πa_o^2. The cross sections for the reverse reactions (88) and (89) were computed from the equilibrium constant using the principle of detailed balancing.

Because of: (1) the apparent insensitivity of the electron density and most of the ion specie densities to changes in the charge exchange rates, (2) the extreme paucity of experimental data and the uncertainty in the charge exchange rates, and (3) the enormous increase in complexity of the expressions for electron and ion densities, the effect of charge exchange will not be included in the present analysis. However, if more sufficient experimental data on charge exchange reactions become available, then Eq. (71) for the rate of production of each ion species should be modified to include these effects:

$$
\begin{aligned}
\frac{d(NO^+)}{dt} = \; & k_{f70.1}(N)(O) + k_{f70.6}(NO)N_e - k_{r70.1}(NO^+)(N_e) \\
+ \; & k_{f88.4}(N_2^+)(NO) - k_{r88.4}(N_2)(NO^+) + k_{f88.7}(N^+)(NO) \\
- \; & k_{r88.7}(N)(NO^+) + k_{f88.9}(O^+)(NO) - k_{r88.9}(O)(NO^+) \\
+ \; & k_{f88.10}(O_2^+)(NO) - k_{r88.10}(O_2)(NO^+) + k_{f89.2}(N_2^+)(O) \\
- \; & k_{r89.2}(N)(NO^+) + k_{f89.4}(N_2)(O^+) - k_{r89.4}(N)(NO^+) \\
+ \; & k_{f89.5}(N_2^+)(O_2) - k_{r89.5}(NO)(NO^+) + k_{f89.6}(O_2^+)(N_2) \\
- \; & k_{r89.6}(NO)(NO^+) + k_{f89.8}(O_2^+)(N) - k_{r89.8}(O)(NO^+) \\
+ \; & k_{f89.10}(N^+)(O_2) - k_{r89.10}(O)(NO^+) \quad .
\end{aligned}
\tag{90}
$$

The expressions for the rate of production of the ionic species N_2^+, O^+, and N^+ including the effects of charge exchange are as complex as the equation for NO^+ [Eq. (90)]. They may be obtained from inspection of Eqs. (70), (88), and (89).

3.5 Effects of Electron Attachment

There are primarily four types of electron attachment processes:

(1) Radiative attachment

$$e + x \rightarrow x^- + h\nu$$

(2) Three-body attachment

$$e + x + y \rightarrow x^- + y$$

(3) Dielectronic attachment

$$e + x \rightarrow (x^-)*$$

(4) Dissociative attachment

$$e + xy \rightarrow x^- + y \quad .$$

Of the various constituents of high-temperature air, O, O_2, and NO have the largest electron affinity (Massey[66]). The electron affinity of O is 1.465 ev; of O_2, 0.43 ev; of NO, small but positive; of N_2, negative; of N, negative. In air at thermal energies of about 0.5 ev corresponding to translational temperatures of about 5000°K, none of these negative ions will be stable. Hence the effects of electron attachment will be neglected.

4. THE FIELD DISTRIBUTION AND THE REFLECTION AND TRANSMISSION COEFFICIENTS OF AN INHOMOGENEOUS, NONLINEAR PLASMA SLAB

The previous sections of this report have been devoted to a detailed description of the method for computing changes in the electron density, electron collision frequency, and electron temperature which have been brought about by the presence of a high-power electromagnetic wave in the ionized flow field surrounding a hypersonic re-entry vehicle. The analysis of the transmission characteristics of the re-entry sheath under the influence of high-intensity microwave radiation is based upon the model of a plane monochromatic wave incident at an arbitrary angle upon a non-uniform multicomponent plasma slab. Part of the complexity of the problem arises from the fact that the reflection and transmission coefficients of the plasma slab depend upon the conductivity of the ionized gas, which is a function of the electron density and collision frequency. These parameters depend upon the electron temperature distribution in the plasma slab. The electron temperature is determined, in turn, by solving the energy balance equation which contains the local field as a parameter. The technique developed for the computation of the transmission and reflection coefficients of the plasma slab consists in replacing the layer by a stack of homogeneous sheets. This model lends itself, in a natural way, to the application of step-by-step numerical integration of the field equations expressed in the form of difference equations. By assuming a value for the amplitude of the transmitted field, and taking backward differences through the slab, the coupled system of equations which describe the conductivity, electron density, energy balance of the electron gas, and the electromagnetic field distribution may all be solved simultaneously. This method is general enough to incorporate the inhomogeneities of electron density and collision frequency into the analysis. The method is not restricted to cases where the permittivity gradient is small compared to a wavelength, such as the convenient WKB asymptotic solution. It is found that for the case of a plane wave of arbitrary polarization, incident at an arbitrary angle upon the slab, the wave may not be entirely uncoupled into two waves: one polarized perpendicular and a second parallel to the plane of incidence. This is because the dielectric constant for a point in the plasma slab is a function of the amplitude of the total local field.

A homogeneous plasma slab does not constitute a physically realistic model for problems concerned with the nonlinear interaction of microwave radiation with the plasma sheath. This is because the microwave power absorbed by the plasma depends critically on the ratio (ω_p/ω). Klein et al.[69] have shown that a maximum transfer of energy between an electromagnetic field and plasma occurs at a point just behind the plasma resonant density $(N_p = \omega^2 m\epsilon_o/e^2)$. This implies, of course, that the interaction of the microwave field with an inhomogeneous plasma results in a nonuniform deposition of energy.

Bloembergen and Pershan[67] have already investigated a somewhat similar problem concerning the solution to Maxwell's equations at a plane interface between a linear and nonlinear medium. However their analysis proceeds from a relationship between the nonlinear atomic properties of a medium and a time dependent susceptibility tensor. In the case of a plasma under the influence of high-power electromagnetic radiation, this corresponds to the situation when the relaxation time $(\tau_{EN} = 1/G\nu)$ for energy transfer between the electron gas and neutral gas is much <u>less</u> than the period of the impressed field $(\tau_{EN} \ll T$ or $\omega \ll G\nu)$. Then the electron temperature will not reach a steady-state value independent of time. Rather, the electron temperature will have a sinusoidal time dependence at the second harmonic of the frequency of the impressed field (see Ginzburg and Gurevich[8]). The electron density will also be time dependent. The characteristic time for electron density variations will be a complicated function of the rates of electron production and electron recombination. Molmud[68] has considered inducing time-dependent perturbations of the electron density and collision frequency in the D-region of the ionosphere by utilizing high-powered ground-based radio transmitters. The perturbations may be observed by the changes produced in the absorption of a second or wanted wave. Here, photoionization, which is the dominant electron production mechanism, is assumed to proceed at a rate which is independent of the electron temperature. The chief electron loss mechanism is the temperature sensitive process of attachment to O_2.

The time-dependent perturbation treatment of nonlinear media of Bloembergen and Pershan[67] predicts harmonic wave generation at the boundary between a linear and nonlinear medium. A reflected second harmonic wave will propagate in the linear medium in the same direction as the reflected fundamental wave, whereas the transmitted harmonic will in general propagate in a direction different than the transmitted fundamental. The matching of the field amplitudes at the boundary for a wave incident at an arbitrary angle leads to generalizations of the usual Fresnel formulas. The treatment of Bloembergen and Pershan[67] is applicable to plane wave transmission and reflection from magnetoactive plasma-vacuum interfaces for incident waves whose frequency is such that $\omega \ll G\nu$. In the present analysis, only waves whose frequency satisfies the condition $\omega \gg G\nu$ will be considered, so that harmonic generation will not occur.

4.1 The Field Equations (MKS)

The propagation of electromagnetic waves in a nonlinear plasma are governed by the usual equations of phenomenological electrodynamics, that is, Maxwell's equations:

$$\vec{\nabla} \times \vec{E} = -\mu_o \frac{\partial \vec{H}}{\partial t} = -j\mu_o \omega \vec{H}$$

$$\vec{\nabla} \times \vec{H} = \epsilon_o \frac{\partial \vec{E}}{\partial t} + \vec{J} = j\epsilon_o \omega \vec{E} + \vec{J} \tag{91}$$

where the diamagnetic effects of the plasma have been ignored. The $\vec{E}$ and $\vec{H}$ fields are assumed to be proportional to $e^{+j\omega t}$. If there is no external magnetic field present in the plasma, the macroscopic current density may be related to the electric field through the scalar conductivity;

$$\vec{J} = \sigma \vec{E} \quad . \tag{92}$$

For the nonlinear medium under consideration, the conductivity depends upon the square of the electric field amplitude. Equation (65) is a general expression for the scalar conductivity in terms of the basic plasma parameters such as electron-electron collision frequencies, the Maxwellian electron velocity distribution function, and the electron density. It is more convenient to work with the complex dielectric constant of the plasma, K, defined in terms of the conductivity by Eq. (66). Maxwell's equations may be written in terms of the complex dielectric constant:

$$\vec{\nabla} \times \vec{E} = -j\mu_o \omega \vec{H}$$
$$\vec{\nabla} \times \vec{H} = j\epsilon_o \omega K \vec{E} \quad . \tag{93}$$

For the range of parameters of interest ($\omega^2 >> \nu^2$, $\omega >> \nu_{ee}$), the approximate expression (67) for the dielectric constant is adequate.

4.2 Wave Propagation in Nonlinear Inhomogeneous Media

4.2.1 A LINEAR, HOMOGENEOUS SEMI-INFINITE PLASMA

Graf and Bachynski[70] have treated the problem of plane wave reflection and transmission from a plane interface between vacuum and a homogeneous plasma. The presence of the electromagnetic field is assumed not to change the properties of the plasma medium. The plasma is characterized by a complex dielectric coefficient:

$$K = K_r - j K_i \; .$$

The vector Helmholz equation which governs wave propagation in the plasma medium is obtained by taking the curl of the first Eq. (93) and substituting curl $\vec{H}$ from the second Eq. (93):

$$\text{curl curl } \vec{E} = \text{grad div } \vec{E} - \nabla^2\vec{E}$$
$$= -j\omega\mu_o \text{ curl } \vec{H} = k_o^2 K\vec{E} \quad . \tag{94}$$

By utilizing the condition:

$$\vec{\nabla} \cdot \vec{D} = \epsilon_o \vec{\nabla} \cdot K\vec{E} = \epsilon_o K \vec{\nabla} \cdot \vec{E} + \epsilon_o \vec{E} \cdot \vec{\nabla} \vec{K} = 0 \tag{95}$$

together with the fact that the plasma is uniform: $\nabla \vec{K} = 0$, Eq. (94) becomes:

$$\nabla^2\vec{E} + k_o^2 K\vec{E} = 0 \tag{96}$$

where $k_o = \omega / c$.

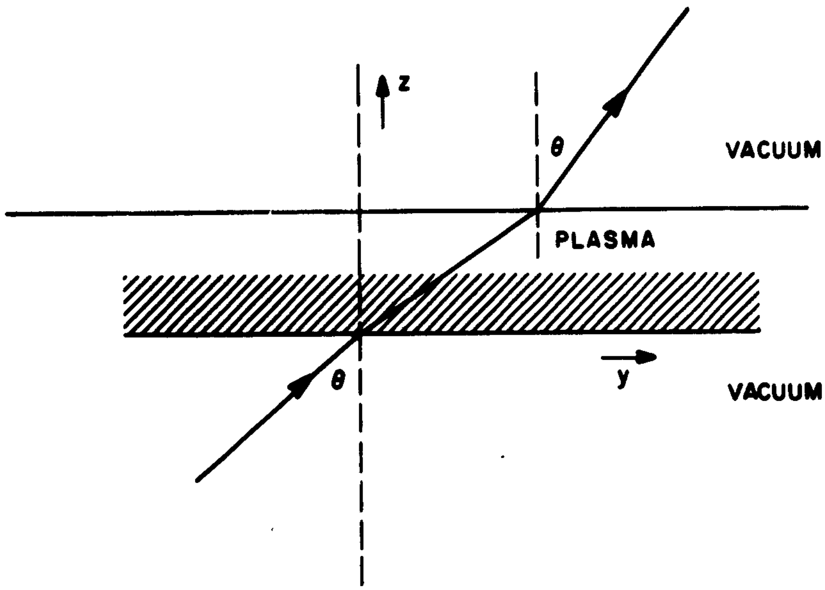

Figure 1.

Figure 1 illustrates the geometry of the problem. Since the field vectors will not vary as a function of x (parallel to the plane of incidence), and the phase velocity in the y direction will be the same in both media, solutions of the wave Eq. (96) for each component may be written:

$$E_i = A_i \exp [PZ + j\,QZ - j\,k_o \sin\theta\, y]$$

where i = x, y, z.

Graf and Bachynski[70] have obtained expressions for the real and imaginary part of the propagation constant in terms of the dielectric constant and the angle of incidence:

$$P = k_o \left[\frac{1}{2}\left\{(K_r - \sin^2\theta)^2 + K_i^2\right\}^{1/2} - \frac{1}{2}(K_r - \sin^2\theta)\right]^{1/2}$$

$$Q = k_o \left[\frac{1}{2}\left\{(K_r - \sin^2\theta)^2 + K_i^2\right\}^{1/2} + \frac{1}{2}(K_r - \sin^2\theta)\right]^{1/2}. \qquad (97)$$

The surfaces of constant phase do not coincide with the surfaces of constant amplitude in the plasma medium. The surfaces of constant amplitude are planes parallel to the interface, whereas the planes of constant phase are at an angle $(\tan^{-1} k_o \sin\theta/Q)$ with respect to the interface.

If Maxwell's Eq. (93) is written out in component form in Cartesian coordinates, it is easily seen that E_x, H_y, and H_z are independent of E_y, E_z, and H_x. The first set of field components corresponds to a wave with the electric vector perpendicular to the plane of incidence and is termed the transverse electric mode (TE). The second set of components corresponds to a wave with the electric vector in the plane of incidence and is termed the transverse magnetic mode (TM). An incident wave polarized at an arbitrary angle may be resolved into these two modes. For a linear homogeneous medium, the propagation characteristics of one of these modes in uninfluenced by the presence of the other mode. However these two modes are not uncoupled for a nonlinear medium, and are uncoupled only in special cases for inhomogeneous media.

Graf and Bachynski[70] have derived generalized Fresnel equations for each mode of polarization by matching the tangential components of $\vec{E}$ and $\vec{H}$ and the normal component of $\vec{D}$ at the interface. Graf and Bachynski[70] have derived expressions for the Poynting vector in the plasma for both modes of polarization. It is demonstrated that for the TE mode the instantaneous Poynting vector changes magnitude and direction during one cycle, so that the wave is not, in general, plane. The average Poynting vector is perpendicular to the planes of constant phase. For the TM mode, it is shown that, in general, neither the instantaneous nor the average Poynting vector is perpendicular to the planes of constant phase. Only for the case of a lossless plasma or normal incidence are the waves in the plasma plane.

For the case of a nonlinear plasma slab with a permittivity gradient perpendicular to the plane interfaces, wave propagation may be considered by imagining the medium to be composed of a stack of linear homogeneous plasma sheets. The solutions of the field equation for a particular sheet will then be representable in the form:

$$E_i = A_i \exp[PZ + jQZ - jk_o \sin\theta\, y] \quad .$$

The essential point is that the real and imaginary parts of the propagation constant $(P + jQ)$ will depend upon the magnitude of the dielectric constant (K) at the point Z in the plasma. The dielectric constant will depend both on the nature of the inhomogenieties and on the square of the local field amplitude.

4.2.2 EXACT SOLUTIONS OF THE WAVE EQUATION FOR A ONE-DIMENSIONAL INHOMOGENEOUS LINEAR PLASMA MEDIUM

A rather small number of exact solutions of the wave equation have been obtained for normal incidence on an interface when the dielectric constant varies only in a direction perpendicular to the plane interface. With reference to Figure 1, exact solutions exist only when the dielectric constant K varies with Z in certain specified fashions. The solutions are usually expressible in terms of tabulated functions, and are particularly useful when the geometric optics approximation $\nabla K/K << 2\pi/\lambda$ is not valid. The geometrical optics approximation is inappropriate when $K \approx 0$.

Ginzburg[17] has discussed the solutions of the wave equation for normal incidence when the dielectric coefficient has a variation in one direction. The form of the dielectric constant and the solutions to the wave equation are as follows:

(1) A linear layer without absorption, $K = 1 - Z/Z_1$, solutions in terms of Bessel functions of one-third order, or Airy functions.

(2) An absorbing linear layer, $K = 1 - Z/Z_1 - j(\alpha + \beta\, Z/Z_1)$, solutions in terms of Bessel functions of one-third order with complex arguments.

(3) A parabolic layer without absorption $K = 1 - \omega_k^2/\omega^2\,(1 - Z^2/Z_m^2)$, where ω_k is the plasma frequency corresponding to the maximum electron density and Z_m is the half-width of the layer, solutions in terms of Weber functions, or parabolic cylindrical functions (see Whittaker and Watson[74]).

(4) A layer characterized by a dielectric constant of the form $K = a(b + Z)^{-2}$, solutions in terms of polynomials in Z.

(5) A layer characterized by a dielectric constant of the form $K = a + b\,e^{\gamma Z} \cdot (1 + e^{\gamma Z})^{-1} + c\,e^{\gamma Z}(1 + e^{\gamma Z})^{-2}$ solutions in terms of hypergeometric functions (see Ref. 74).

Pappert and Plato[71] have treated the cases of a linear profile, a parabolic profile, and a cosine profile. A cosine profile in dielectric constant leads to solutions of the wave equation in the form of Mathieu functions. Taylor[72] has demonstrated that the solutions to the wave equation may be represented in terms of cylindrical Bessel functions for an exponential profile and for a profile varying as Z^{-2}. Buchsbaum[73] has obtained solutions of the wave equation in the form of Hankel functions for a dielectric constant of the form

$$K = K_o\,(1 + \alpha Z)^n \quad .$$

It should be mentioned that there exist cases where an exact solution to the wave equation can be obtained for normal incidence and that these cases are capable of being extended to arbitrary angles of incidence for the TE mode of polarization. This is always true when an exact solution to the wave equation has been obtained corresponding to a dielectric constant K(z), such that it contains an arbitrary constant that does not depend upon Z. The proof of this statement rests upon the following three facts:

(1) For the TE mode of polarization, $\vec{E} \cdot \overline{\nabla}\vec{K}(Z) = 0$, so that, by Eq. (95), $\vec{\nabla} \cdot \vec{E} = 0$.

(2) The component of the phase velocity of the incident wave parallel to the interface ($k_o \sin \theta$) is the same as the component of the phase velocity of the transmitted wave parallel to the interface.

(3) The formal structure of the wave Eq. (96)

$$\frac{d^2E}{dZ^2} + k_o^2 (K - \sin^2\theta) E = 0$$

is not changed for the TE mode as the angle of incidence (θ) is varied.

4.2.3 APPROXIMATE METHODS FOR OBTAINING SOLUTIONS TO THE WAVE EQUATION FOR A ONE-DIMENSIONAL INHOMOGENEOUS LINEAR PLASMA MEDIUM

Most treatments of electromagnetic wave propagation in inhomogeneous media assume that the variations of dielectric constant occur along only one coordinate direction. Studies of electromagnetic wave transmission and reflection characteristics from plane media whose dielectric constant varies in a direction perpendicular to the plane interfaces have been conducted by Schelkunoff,[75] Penico,[12] Stickler,[13] Richmond,[14] Albini and Jahn,[77] Nicoll and Basu,[78] and Brekhovskikh.[79] Klein et al.[69] and Safran and Meltz[80] have applied the WKB asymptotic approximation and a numerical integration of the Riccati equation for the complex impedance of a dielectric slab to the problem of computing the transmission characteristics of the inhomogeneous re-entry plasma sheath. In problems involving propagation between two media with differing dielectric constant, the surface at which the discontinuity occurs usually may be represented as a plane, cylindrical, or spherical boundary. A general problem would involve obtaining solutions to the wave Eq. (94) to compute physical observables, such as phase shift, absorption coefficients, reflection and transmission coefficients. Schelkunoff[75] has pointed out that for the general case of waves propagating in an inhomogeneous medium, such that the direction of propagation coincides with the direction of the gradient of dielectric coefficient, it is not always possible to interpret the solutions of the wave Eq. (94) as outgoing or incoming disturbances. However, for the present analysis, in which a plane-layered plasma slab is bounded on both sides by vacuum, this difficulty will not arise except perhaps when the incident wave is polarized in the plane of incidence and the dielectric constant $K \approx 0$. This would correspond to the physical situation where plasma waves could be excited in the ionized medium.

The general form of the Maxwell Eqs. (93) indicates that the electric field components are coupled. These field components become uncoupled only for very special coordinate geometries. Penico[12] has investigated the uncoupling of the field components for several orthogonal curvilinear coordinate systems. The choice of coordinate system is usually dictated by the nature of the boundary surface when more than one dielectric medium is involved. This facilitates the matching of the field components at the boundary. Penico[12] has emphasized the fact that the usual techniques for obtaining solutions to the wave Eq. (94) require that two of the three unknown field components be eliminated from the equation, so that a scalar equation for a single field component may be derived. Also, the operator ∇^2 which appears in Eq. (94) may be interpreted as the Laplacian only for Cartesian coordinates (see Stratton,[76] p. 49). Penico[12] claims that the reduction of the wave Eq. (94) to a scalar equation for a single field component is possible in general only when the dielectric constant K is a function of one coordinate.

Consider the oblique incidence of a monochromatic plane wave on a plane-layered medium which is inhomogeneous in a direction normal to the interface (Z-axis). Outside the layer, for $Z < O$ (see Figure 1), the incident wave may be written:

$$E = E_o \exp[\,-jk_o\,(\sin\theta\, y + \cos\theta\, Z)]\,. \qquad (98)$$

Since the incident wave is independent of the x-coordinate, both reflected and refracted wave will be independent of the x-coordinate for a medium in which the dielectric constant is independent of x. Using Gauss' law, Eq. (95) may be written in Cartesian coordinates in the form:

$$\frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial Z^2} + k_o^2 K E_y + \frac{\partial}{\partial y}(\vec{E}\cdot\overrightarrow{\nabla \ln} K) = 0$$

$$\frac{\partial^2 E_Z}{\partial y^2} + \frac{\partial^2 E_Z}{\partial Z^2} + k_o^2 K E_Z + \frac{\partial}{\partial Z}(\vec{E}\cdot\overrightarrow{\nabla \ln} K) = 0 \qquad (99)$$

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial Z^2} + k_o^2 K E_x = 0\,.$$

The field components not only satisfy the wave Eq. (99) they must also satisfy Maxwell's Eq. (93), which assume the following form in Cartesian coordinates:

$$\frac{\partial E_y}{\partial Z} - \frac{\partial E_Z}{\partial y} = -j\mu_o \omega H_x$$

$$-\frac{\partial E_x}{\partial Z} = -j\mu_o \omega H_y$$

$$\frac{\partial E_x}{\partial y} = -j\mu_o \omega H_Z \qquad (100)$$

$$\frac{\partial H_y}{\partial Z} - \frac{\partial H_Z}{\partial y} = j\epsilon_o \omega K E_x$$

$$-\frac{\partial H_x}{\partial Z} = j\epsilon_o \omega K E_y$$

$$\frac{\partial H_x}{\partial y} = j\epsilon_o \omega K E_Z .$$

By examining Eq. (100), it may be clearly seen that the vector components E_y, E_z, H_x form one independent set; whereas the components E_x, H_y, and H_z form a second independent set. The first set corresponds to the TM mode of polarization; the second, the TE mode of polarization. An incident transverse electric mode will excite only a TE mode in the plasma. Similarly an incident TM wave will excite only a TM mode in the plasma slab. This is true for a nonlinear one-dimensional inhomogeneous medium. Moreover for a linear inhomogeneous medium, the propagation of the TE wave is completely independent of the presence of the TM wave, so that the modes are entirely uncoupled. However, for a nonlinear medium, whether it is inhomogeneous or not, the propagation characteristics of the TE wave depend upon the TM mode, for the dielectric coefficient is a function of the square of the total field amplitude.

The phase velocity of the electromagnetic wave in the y-direction (parallel to the interface) is the same in vacuum as in the plasma slab, even when the medium is nonlinear and inhomogeneous. This is merely a statement of Snell's law of refraction, and may easily be proved by considering the nonlinear inhomogeneous medium as the limiting case of a medium composed of a stack of linear homogeneous layers:

$$k_o \sin\theta = k \sin\theta_r$$

where

$$k_o = \frac{\omega}{c}\ , \quad k = k_o\sqrt{K} \quad .$$

θ = angle of incidence and θ_r is the angle of refraction at the first interface. One important consequence of Snell's Law is that:

$$\frac{\partial}{\partial y} E_i = - j\, k_o \sin\theta\, E_i$$

where i = x, y, or z. Another consequence is that the angle of incidence equals the angle at which the transmitted wave emerges from the far face of the inhomogeneous nonlinear slab.

Richmond[14] has set up the field equations in the form of difference equations so that the field distribution in one-dimensional inhomogeneous slabs may be computed using a step-by-step numerical integration technique. By assuming the value for the amplitude of the transmitted wave on the source-free side of the slab, the field distribution in the medium may be calculated by taking backward differences. For the TE mode of polarization, Richmond[14] has utilized the wave Eq. (99) for the x-component of the electric field. To obtain the solution to the difference equation, the field must be known at two points. The amplitude of the transmitted wave at the source-free side of the slab, corresponding to the point Z = d, is assumed. The field at the location of the first backward increment, corresponding to Z = d-h, is obtained by making a Taylor series expansion of the field about the point z = d in powers of the increment size h. The difference equation form of the wave Eq. (99) then prescribes the field amplitude at the point z = d-2h in terms of the field amplitudes at the points z = d and z = d-h. Each backward step corresponds to the numerical integration of the wave equation across a layer whose dielectric constant K(z) is uniform over the step-size h. The boundary conditions for the field components must be satisfied at each transition across a discontinuity in dielectric constant. The matching of the field components at each boundary of the plane-layered medium presents no difficulty for the TE mode, since the tangential components E_x and H_y and the normal component H_z are all continuous across a discontinuity in dielectric constant K. The treatment for the TM mode is somewhat different. Here there is a component of the electric field E_z, which is normal to the plane interfaces. Since the normal component of the electric displacement must be continuous across a discontinuity in dielectric constant, the boundary condition for the D vector would require that the condition

$$K(Z_+)\, E_{Z_+} = K(Z_-)\, E_{Z_-} \tag{101}$$

be satisfied at each step in the numerical integration, where the plus subscript refers to the layer just to the right of the discontinuity and the minus subscript refers to the layer at the left. In the case of a linear inhomogeneous medium, this added difficulty may be circumvented by considering only the field component tangent to the plane interfaces, E_y. Once the field distribution for the tangential component E_y is determined, there is information sufficient to determine the total reflection and transmission coefficients of the slab. This is because the angle of incidence must equal the assumed angle of transmission. However, for a nonlinear medium, the case of the TM mode of polarization presents special difficulties. It is impossible to find the field distribution for the E_y component alone, because all the electric field components are coupled through the dielectric coefficient K, where K is a function of all the components. For a nonlinear plane-layered medium, the step-by-step numerical integration must be carried out for all the electric field components simultaneously, with the condition of Eq. (101) imposed at each dielectric coefficient discontinuity for the normal electric field component E_z. This involves an iteration at each boundary, since the dielectric coefficient to the left of each discontinuity, $K(z_-)$, is unknown. First, the value that $K(z_-)$ would have if the medium were linear is assumed, and this determines E_{z-}. The fields thus obtained are used to compute a new value for $K(z_-)$, and then a new value for E_{z-} is obtained from Eq. (101). Convergence of the iterative procedure should be rapid since the nonlinear departures from the linear case will be relatively small.

4.2.4 TECHNIQUE FOR COMPUTING THE FIELD DISTRIBUTION, TRANSMISSION AND REFLECTION COEFFICIENTS OF A NONLINEAR ONE-DIMENSIONAL INHOMOGENEOUS PLASMA SLAB

The procedure used to compute the field distribution in an inhomogeneous nonlinear plasma slab is similar to the method used by Richmond[14] for the case of linear inhomogeneous media. A major modification involves the direct use of Maxwell's equations, Eq. (100), to find the field distribution, instead of working with the wave Eq. (99). This facilitates the numerical integration, for a Taylor series expansion of the field amplitude does not have to be made in order that the numerical integration be started.

Equation (100) may be written in dimensionless form:

$$\begin{aligned} \frac{\partial u_y}{\partial \tau} + j \sin\theta\, u_z &= -j\, v_x \\ \frac{\partial u_x}{\partial \tau} &= j\, v_y \end{aligned} \tag{102}$$

$$\sin\theta\, u_x = v_z$$

$$\frac{\partial v_y}{\partial \tau} + j \sin\theta\, v_z = K u_x \qquad (102)$$

$$\frac{\partial v_x}{\partial \tau} = -j K u_y$$

where

$$\sin\theta\, v_x = -K u_z$$

$$u_i = \sqrt{\epsilon_o}\, E_i$$

$$v_i = \sqrt{\mu_o}\, H_i$$

$$\tau = k_o Z \quad .$$

Here the dielectric constant K is a function of the electron density N_e and effective collision frequency ν_{EFF}. For the present analysis the approximate expression (67) for K is adequate. At each step in the numerical integration, the energy balance equation, Eq. (75), must be solved for the electron temperature T_e. This permits the determination of the electron density from Eq. (72) and the effective collision frequency from Eq. (2). The field amplitude squared, which appears in the ohmic heating term of the energy balance Eq. (75), is the square of the total field:

$$E^2 = E_x^2 + E_y^2 + E_z^2 \quad .$$

The solution is started by assuming values for the amplitude of the transmitted field u, the angle of polarization ϕ [$\tan\phi = (u_y^2 + u_z^2)^{+1/2} u_x^{-1}$], and the angle of transmission θ($\theta = \tan^{-1} u_z u_y^{-1}$) at the source-free side of the slab $z = d_+$, where d = slab thickness and plus subscript refers to a location on the side of a dielectric constant discontinuity furthest from the source. At a point just inside the plasma slab, corresponding to $z = d_-$, the tangential electric and magnetic fields u_x, u_y, v_x, v_y, and the normal magnetic field v_z have the same magnitude as the assumed values at the point $z = d_+$. If there is a normal component of electric field u_z, its value at a point just inside the slab (u_{d-}) must be obtained by making successive approximations to the solution of Eq. (101),

$$u_{d_-} K(d_-) = u_{d_+} K(d_+) = u_{d_+} \quad ,$$

where the first approximation to $K(d_-)$ may be taken as the value of the dielectric constant for a linear medium. If there is no component of the electric field normal to the interfaces (TE mode), there is no change in the value of the amplitude of any of the field components as a discontinuity in dielectric constant is traversed. However, whether the mode is TE, TM, or mixed, the calculation of the dielectric constant at the point $z = d_-$ involves the solution of the energy balance Eq. (75) for the electron temperature, the determination of the electron density from Eq. (72), and the determination of the effective collision frequency from Eq. (2). Once a self-consistent set of values have been obtained for the field components and dielectric constant at the point $z = d-$, Maxwell's equations expressed in the form of difference equations give a prescription for finding the fields at the point $z = (d-h)_+$, where h is the increment size used in the difference equation. The distance h corresponds to a plane layer thickness over which the electron temperature, density, and collision frequency are assumed to remain constant. The dimensionless form of Maxwell's equations, Eq. (102), expressed as difference equations are:

$$\begin{aligned}
u_y(\tau-h) &= u_y(\tau) + jh\,[\sin\theta - \frac{K}{\sin\theta}]\, u_z(\tau) \\
u_x(\tau-h) &= u_x(\tau) - jh\, v_y(\tau) \\
v_z(\tau-h) &= \sin\theta\, u_x(\tau-h) \\
v_y(\tau-h) &= v_y(\tau) + jh\,[\sin^2\theta - K]\, u_x(\tau) \\
v_x(\tau-h) &= v_x(\tau) + jK\, u_y(\tau) \\
u_z(\tau-h) &= -\sin\theta\, K^{-1}\, v_x(\tau-h)\,.
\end{aligned} \tag{103}$$

Equations (103) yield the values of the field amplitudes at the point $z = (d-h)_+$, once the fields at the point $z = d_-$ are known. All the field components at the point $z = (d-h)_+$ are equal to the fields at the point $z = (d-h)_-$, except the normal component of the electric field, u_z, which is again determined by obtaining a self-consistent set of solutions to Eq. (101):

$$K(\tau-h)_-\, u_z(\tau-h)_- = K(\tau-h)_+\, u_z(\tau-h)_+$$

Eq. (67) for $K(\tau-h)_-$, the energy balance Eq. (75), the electron density Eq. (72), and the Eq. (2) for effective collision frequency. If Maxwell's equations [Eq. (103)] are combined with the boundary condition for the normal electric-field component, the results may be represented in the form:

$$
\begin{aligned}
u_y(\tau-h) &= u_y(\tau) + jh[\sin\theta - \frac{K}{\sin\theta}]\, u_z(\tau)_- \\
u_z(\tau-h)_- &= \frac{K(\tau-h)_+}{K(\tau-h)_-}\, u_z(\tau-h)_+ \\
u_z(\tau-h)_+ &= -\frac{\sin\theta}{K}\, v_x(\tau-h) \\
v_x(\tau-h) &= v_x(\tau) + j\,K\,u_y(\tau) \\
u_x(\tau-h) &= u_x(\tau) - j\,h\,v_y(\tau) \\
v_z(\tau-h) &= \sin\theta\, u_x(\tau-h) \\
v_y(\tau-h) &= v_y(\tau) + jh[\sin^2\theta - K]\, u_x(\tau) \quad .
\end{aligned}
\tag{104}
$$

The difference equation for the normal component of the electric field is singular when $K = 0$. This situation corresponds to the excitation of plasma waves in the medium by the component of the electric field normal to the interface (TM mode). If collisions are present the energy associated with these plasma waves will result in heating of the plasma (see Ginzburg[17]).

The step-by-step numerical integration is performed until the point $z = 0_-$ is reached, corresponding to the face of the slab upon which the plane wave is incident. Once the entire field distribution has been obtained, the reflection and transmission coefficients for each mode of polarization may be found immediately from the following relations:

Transverse Electric Mode:

$$
\begin{aligned}
\sin\theta\,(|\vec{H}_{inc}| + |\vec{H}_R|) &= -H_z(o) \\
\cos\theta\,(|\vec{H}_{inc}| - |\vec{H}_R|) &= H_y(o)
\end{aligned}
\tag{105}
$$

$$R_{\perp} = \frac{|\vec{H}_R|}{|\vec{H}_{inc}|} = \frac{-H_z(o)\cos\theta - H_y(o)\sin\theta}{-H_z(o)\cos\theta + H_y(o)\sin\theta}$$

$$T_{\perp} = \frac{|\vec{H}_T|}{|\vec{H}_{inc}|} = \frac{\sin 2\theta\,[H_y^2(d) + H_z^2(d)]^{1/2}}{[-H_z(o)\cos\theta + H_y(o)\sin\theta]}$$

Transverse Magnetic Mode: (105)

$$\sin\theta\,(|\vec{E}_{inc}| + |\vec{E}_R|) = E_z(o_-)$$

$$\cos\theta\,(|\vec{E}_{inc}| - |\vec{E}_R|) = -E_y(o)$$

$$R_{||} = \frac{|\vec{E}_R|}{|\vec{E}_{inc}|} = \frac{E_z(o_-)\cos\theta + E_y(o)\sin\theta}{[E_z(o_-)\cos\theta - E_y(o)\sin\theta]}$$

$$T_{||} = \frac{|\vec{E}_T|}{|\vec{E}_{inc}|} = \frac{\sin 2\theta\,[E_z^2(d_+) + E_y^2(d)]^{1/2}}{[E_z(o_-)\cos\theta - E_y(o)\sin\theta]}$$

where

$|\vec{H}_{inc}|$ and $|\vec{E}_{inc}|$

are the magnitudes of the incident magnetic and electric vectors, $|\vec{H}_R|$ and $|\vec{E}_R|$ are the magnitudes of the reflected magnetic and electric vectors, $|\vec{H}_T|$ and $|\vec{E}_T|$ are the magnitudes of the transmitted magnetic and electric vectors. $R_{\perp}$ and $T_{\perp}$ refer to the reflection and transmission coefficients for the TE mode and $R_{||}$ and $T_{||}$ refer to these quantities for the TM mode. The angle θ is the angle which was assumed as the angle of transmission. It should be noted that the angle of transmission must equal the angle of incidence.

In this analysis, calculations will be performed only for normal incidence. In this case, Eq. (105) assumes an indeterminate form. For normal incidence there is no distinction between the TE mode and the TM mode. The following relations should be used for computing the reflection and transmission coefficients of a non-linear slab for normal incidence:

$$R = \frac{u(o) - v(o)}{u(o) + v(o)}$$

$$T = \frac{u(d)}{u(o)} \tag{106}$$

$$E_{inc} = \frac{u(o)}{(1+R)} \ (3.37 \times 10^5) \text{ volts/m}$$

$$E_R = R \frac{u(o)}{(1+R)} (3.37 \times 10^5) \text{ volts/m} \ .$$

The computations may be performed for several step sizes (h) to obtain an estimate of the accuracy of the solution. Stickler[13] and Richmond[14] have discussed the dependence of the accuracy of the solution on the increment size for the case of transmission through plane-layered dielectric slabs.

5. CONCLUSIONS

The results of the computations for the reflection and transmission coefficients of a nonlinear plasma slab which simulates the composition of high-temperature air (5000°K) will be made available in a future report. Order of magnitude estimates reveal that power fluxes of the order 100 to 1000 watts/cm^2 are capable of producing factors of four or five changes in electron temperature. This will result in about factors of two change in collision frequency and factors of two or more changes in electron density. From the graphs of attenuation constant vs $(\omega_p/\omega)^2$ for various ratios of ν/ω presented by Bachynski et al.,[31] it may be inferred that these changes will induce changes in absorption coefficient of factors of two or more for certain ranges of the parameters (ω_p/ω) and (ν/ω). These effects are certainly important from a re-entry communications system point of view.

Unfortunately the order of magnitude changes which are expected to result from this type of nonlinear interaction are of the same order as the uncertainties in the rate coefficients for electron-ion recombination. The present state of the art is such that even for calculations of electron density profiles about blunt-nosed re-entry vehicles under equilibrium flow regimes, only order of magnitude estimates may be obtained. Thus, the effects of the nonlinear interaction of microwave radiation with ionized flow fields, as presented in this report, will result in changes of electron density which are of the same order of magnitude as the uncertainties in the electron density. However, as more accurate information on the rate coefficients for the many processes that occur in high-temperature air become available, the theory presented will enable accurate predictions to be made on changes in power transmitted through ionized flow fields at relatively high power levels.

The present analysis has been based upon the assumption that the plasma medium reaches a steady state under the influence of a plane monochromatic electromagnetic wave. The boundaries are assumed to be so steep compared to a wavelength that they may be represented as plane interfaces. Figures 10a, b, and c of Lin and Teare's[4] report indicate that the electron density has essentially reached its maximum value one hundred upstream mean free paths behind a normal one-dimensional shock front for Mach numbers between about 15 and 25. At an altitude of 200,000 ft, one upstream mean free path is about 0.05 cm so that within a distance of 5 cm the electron density builds up to about 5×10^{12} particles/cm^3. If this region over which steep electron density gradients occur is less than about a quarter of a wavelength, they may be represented as plane interfaces to a good approximation. Of course, at altitudes below 200,000 ft the gradients are even steeper; while at about 250,000 ft the shock front becomes diffuse. At 100,000 ft, one hundred upstream mean free paths is about 0.1 cm. The one hundred mean free paths criterion for electron density buildup applies only to a normal one-dimensional shock. The model considered in this report of a planar slab is more appropriate for regions that lie at a distance of more than one nose radii behind the tip of a hemisphere-cylinder body. Naturally the plasma sheath in this region is cylindrical in shape, but a planar geometry will give an accurate description of the electromagnetic reflection and transmission characteristics if the radius of the cylinder is large compared to a wavelength.

Since the boundaries defining the plasma medium at altitudes less than 200,000 ft are very steep compared to wavelengths for X-band radiation (3 cm) and frequencies lower than 10 kMcps, the boundaries will be represented as plane interfaces. For an abrupt discontinuity in dielectric constant of the medium, there is no energy deposition at the discontinuity, only reflection or transmission of electromagnetic radiation. Hence the assumption will be made that the boundaries of the plasma medium are not disturbed by electromagnetic radiation at X-band frequencies for power levels up to 1000 watts/cm^2. This appears to contradict the result of the investigations of King,[16] who found that the plasma-air boundary representing a shock front will move toward a source of high-intensity microwave radiation. This study is based upon the WKB solution of the wave equation for the field distribution in the plasma medium, and is valid only when the electron density gradients are small compared to a wavelength (when $\omega > \omega_p$ and $d \ln N_e/dZ \ll 1/\lambda$). The application of the WKB approximation to problems of propagation through the re-entry plasma sheath appears to be restricted to altitudes above 200,000 ft and to electromagnetic radiation wavelengths of about 1 cm or less. The analysis as presented in this report pertains to electromagnetic propagation from the vehicle out through the re-entry sheath, whereas King is concerned primarily with propagation in the opposite sense. Thus, insofar as propagation out through the re-entry sheath is concerned, the interface of the plasma medium defined by the boundary layer will not physically move because of the proximity of the vehicle surface. Since the power level of the electromagnetic wave will be considerably reduced by the time the wave has traversed the plasma layer, one may assume that the boundary of the plasma medium at the shock front will not move. One of the basic assumptions in this report is that the electron density and temperature profiles will redistribute themselves behind the shock front under the influence of high-power electromagnetic radiation in such a fashion that no mass motion of electrons will result, nor will the boundaries defining the plasma medium be seriously perturbed.

King[16] has found that an ionization front will move toward the source of high-power electromagnetic radiation. This result was obtained under the assumption that the WKB approximation adequately represents the solution to the wave equation in the plasma medium. Thus, King has obtained the result of a moving plasma boundary precisely for the case where the electron density gradients at the boundary vary slowly compared to a wavelength. The model King has chosen to represent the interaction of high-power radio-frequency radiation with a plasma is somewhat idealized, in that a full description of the many microscopic processes is not taken into account. For instance only three parameters are used to characterize the plasma medium: the ionization cross section (σ_i), the total cross section (σ), and the ionization potential (W_i). Not only is such a model restricted to plasmas consisting of but a single neutral specie, but also King uses the same parameters (σ, σ_i, and W_i) to represent the medium in front of and behind the ionization profile. This is certainly not realistic with regard to air shocks, where the constituent NO (which has a low ionization potential) exists behind the shock front but not in the unperturbed air ahead of the shock front. Then, too, the assumption that the quantity

$$\left[\frac{\sigma_i}{W\sigma} \quad \frac{N_e e^2 \nu}{m(\omega^2+\nu^2)} \quad E^2\right]$$

represents the number of ion pairs created per unit time is founded on the supposition that the collision frequency for momentum transfer ν, which appears in the formula for the conductivity, is identical to the 'effective' collision frequency given by Eq. (2). But this is approximately true only if the velocity-dependent cross section for momentum transfer $\sigma(v)$ is not too strongly dependent on velocity and if the isotropic part of the electron distribution function f_o is Maxwellian. King has avoided a discussion of the conditions under which the isotropic part of the electron distribution function will remain Maxwellian and hence there is no direct employment of the electron temperature as a parameter. This was possible by coupling the solution to the wave equation directly to the particle conservation (continuity) equation, by-passing any consideration of an energy balance equation. However the conditions under which the isotropic part of the electron distribution function will remain Maxwellian under the influence of an electromagnetic field ($\nu_{ee} >> G\nu$) are precisely the same conditions that lead to a large thermal conductivity coefficient of the electron gas (K_{ee}). Electron-electron collisions are effective in transporting heat from one part of the electron gas to another, and a high interelectron collision frequency implies a large thermal conductivity coefficient.

These preliminary facts lead to the important observation that the electromagnetic energy deposition in a slowly varying electron density gradient is <u>not always</u> as selective as Klein et al[69] claim, because the pattern of electromagnetic energy deposition in a plasma may be determined, in certain important instances, by the thermal conductivity of the electron gas in addition to its dependence on the variation of dielectric constant of the plasma (due to electron density gradients). Klein et al[69] have found that, for a slowly varying electron density gradient, a maximum transfer of electromagnetic energy to the plasma will occur just anterior to the resonant plasma depth. The resonant plasma depth is the depth of the plasma for which the electron density becomes critical, $N_c = (\omega^2+\nu^2)m\epsilon_o/e^2$. A high interelectron collision frequency ($\nu_{ee} >> G\nu$, a condition that is met for the stagnation region of the re-entry plasma sheath and many radio-frequency plasma discharges in the laboratory) will produce an equilibration within the entire body of the electron gas of the electromagnetic energy deposited at any point on the electron density profile. If $\nu_{ee} >> G\nu$, the time for energy equilibration within the electron gas $\tau_{ee} = 1/\nu_{ee}$ is much faster than the time for energy transfer between electron gas and neutral gas. Electromagnetic energy which is most efficiently deposited at the critical density on the electron density profile will very quickly be redistributed within the entire electron gas behind the ionization front. The effect of the high thermal conductivity of the electron gas, together with the rapid electron-electron thermalization time, has not been included in King's model. It is expected that the electron thermal conductivity effects will tend to reduce the local rate of electron growth at the resonant plasma depth, so that the picture of an ionization front moving toward the source of electromagnetic energy will be modified. It is suggested that the whole phenomenon of moving ionization fronts, including the effects of electron thermal conductivity, should be more intensively investigated both from the experimental and theoretical point of view.

It should be stressed at this point that the important effects of electron gas thermal conductivity may be incorporated into the scheme presented in this report. In the first approximation the effects of the thermal conductivity of the electron gas are neglected. This is justified if the electron temperature gradients established by the presence of the electromagnetic field are not greater than one hundred times the gradients that prevailed in the absence of the electromagnetic field. It may then be easily shown that the terms in the energy balance which represent heat flow from electron gas to neutral and ion gas are one or two orders of magnitude greater than the terms which represent heat transport from one part of the electron gas to another (when $\nu_{ee} >> G\nu$, the term $\vec{\nabla} K_{ee} \quad \vec{\nabla} T_e$ is dominant). Hence, in the first approximation the plasma medium may be divided into a stack of homogeneous slabs with no thermal coupling between them. Then, the steady-state electron density and temperature distribution due to a high-power electromagnetic wave may be found by using the energy balance equation, the equation for the effective dielectric constant of the plasma medium, and Maxwell's equations written in the form of difference equations. The energy balance equation is solved for the electron temperature at each step in the numerical integration, where only terms that represent heat flow from electron gas to neutral and ion gas are included. Once the new electron density and temperature distribution have been found due to the perturbing electromagnetic field, a correction to the temperature distribution due to thermal conductivity may be taken into account by an iterative procedure. Consider a thin sheet of plasma within the slab, located at the position Z and with thickness dZ. The heat flowing into the face at Z is $\int_Z K_{ee}(dT_e/dZ)\, dA$, whereas the heat flowing into the face at Z + dZ is $\int_{Z+dZ} K_{ee}\, (dT_e/dZ)\, dA$. By using the divergence theorem, the net heat flowing into the thin sheet may be written:

$$\frac{d}{dZ} K_{ee} \frac{dT_e}{dZ} = \frac{dK_{ee}}{dT_e} \left(\frac{dT_e}{dZ}\right)^2 + K_{ee} \frac{d^2T_e}{dZ^2} \quad .$$

Since only the heat flow produced by the perturbing electromagnetic field should be considered in the energy balance equation, the quantity

$$\left[\frac{dK_{ee}}{dT_e} \left(\frac{dT_e}{dZ}\right)^2 + K_{ee} \frac{d^2T_e}{dZ^2} - \frac{dK_{ee}}{dT} \left(\frac{dT}{dZ}\right)^2 - K_{ee} \frac{d^2T}{dZ^2}\right]$$

represents the heat flow into a thin sheet of plasma due to electron thermal conductivity.

After finding the temperature distribution in the slab by neglecting the electron thermal conductivity, a second step-by-step numerical integration of the field equations may be performed, where now the term

$$\left[\frac{dK_{ee}}{dT_e} \left(\frac{dT_e}{dZ}\right)^2 + K_{ee} \frac{d^2T_e}{dZ^2} - \frac{dK_{ee}}{dT} \left(\frac{dT}{dZ}\right)^2 - K_{ee} \frac{d^2T}{dZ^2}\right]$$

is added to the energy balance equation at each step in the numerical integration. This procedure will give a new electron temperature distribution (and a new electromagnetic field distribution) that may now be used to perform a third step-by-step numerical integration. The entire procedure may be repeated until a completely self-consistent set of solutions to Maxwell's equations, the energy balance equation, and the equation for the effective dielectric constant of the medium have been obtained.

The assumption that the neutral and electron gas remain stationary during the interval of high-power interaction is reasonable, so that there will be little cooling of the plasma as a result of convective flow. The characteristic time for energy equilibration within the electron gas is $\tau_{ee} = 1/\nu_{ee} \approx 10^{-9}$ sec and the characteristic time for energy transfer between electron gas and neutral gas is $\tau_{EN} \approx 10^{-6}$ sec at $T_e = 5000°K$ and $\tau_{EN} \approx 10^{-7}$ sec at $T_e = 20,000°K$ for the stagnation region of an air shock at 200,000 feet. This relaxation time for energy transfer between electron gas and neutral gas will become as fast as 10^{-7} sec at $T_e = 5000°K$ and 10^{-8} sec at $T_e = 20,000°K$ for shocks at 100,000 feet. Assuming a characteristic length of 10 cm corresponding to the width of an electromagnetic beam, and a maximum flow velocity of 10^6 cm/sec, the time for a portion of the plasma sheath to flow out of the influence of the perturbing beam is approximately 10^{-5} seconds. Thus the very important conclusion has been reached that the plasma sheath may be considered as a stationary gas when considering the perturbing influence of high-power electromagnetic radiation on the ionized flow field, especially at altitudes below 200,000 ft and at power levels that may raise the electron temperature to 20,000°K (corresponding to an electron fractional energy loss $G \approx 10^{-2}$).

The entire analysis has been carried as far as the present state of the art permits. At power levels higher than about 1000 watts/cm^2, electron temperatures will be brought into a range (above 30,000°K) where the cross sections for elastic and inelastic electron-neutral collisions and the rate coefficients for electron-ion recombination are unknown.

A basic parameter in the analysis, the relative fractional energy loss of an electron per collision (G) is known for most constituents only up to electron energies of about 3 ev (30,000°K). Of course, it has been implicitly assumed during this investigation that the values of the relative fractional energy loss of an electron per collision with neutrals which have been experimentally determined for cold gases ($T \approx 300°K$) will still have the same dependence on electron temperature for high temperature air ($T = 5000°K$). In other words it has been assumed that G is only a function of T_e and not T. This is similar to the implicit assumption of Shkarofsky et al[32] that the collision cross sections for electron scattering of the neutral constituents is a function only of T_e and not T. The gas temperature is assumed to have an effect only in that the degree of dissociation of air changes with gas temperature, so that the effective scattering cross section or effective G factor of the gas mixture must be determined by weighting the cross sections or G factors by the relative concentrations of the appropriate neutral constituents. However the cross-section σ_j or the parameter G_j for the particular j^{th} neutral constituent is assumed to vary only with T_e and not T.

That this assumption may not be entirely correct can be seen from the following considerations. A homonuclear diatomic molecule such as N_2 or O_2 has no permanent dipole moment, so that the vibrational levels corresponding to each electronic state of the molecule are metastable and may have a lifetime as long as 10^{-5} seconds. The rotational states have a lifetime of about 10^{-9} seconds. When the translational temperature behind an equilibrium shock in air is about T = 5000°K (0.5 ev), most of the O_2 will be dissociated but there will be an appreciable amount of N_2. Molecule-molecule and atom-molecule collisions will occur at the rate of about 10^{+7} sec^{-1}. It requires only one or two molecule-molecule collisions to achieve translational-rotational equipartition and about 100 molecule-molecule collisions to achieve translational-vibrational equipartition. For T_e = 5000°K, $\rho/\rho_o = 10^{-3}$; and for T = 5000°K, $G\nu \approx 10^6$ sec^{-1}. At about one hundred upstream mean free paths behind a normal one-dimensional shock at 200,000 ft, corresponding to a velocity of about 20,000 ft/sec, an appreciable fraction of the molecules (mainly NO and N_2) will be in excited vibrational states. Harries[28] found that the lowest vibrational state of N_2 is 0.29 ev. Since NO is a nonsymmetric diatomic molecule, it has a permanent dipole moment; thus the lifetime of a vibrational level is expected to be about 10^{-8} sec. The time between electron-neutral inelastic collisions is $\tau_{EN} = 1/G\nu \approx 10^{-6}$ sec, so that electron collisions with the molecule NO can result in collisions which excite at least the lowest vibrational level of NO. However the lifetimes of the vibrational levels of the homonuclear molecules N_2 and O_2 are about 10^{-5} sec and 10^{-3} sec respectively, which are longer than $1/G\nu \approx 10^{-6}$ seconds. Hence electron energy loss under impact with the molecules N_2 and O_2 can occur only by excitation of the rotational levels of the associated metastable vibrational levels of these molecules, and not by excitation of the vibrational levels themselves because the second vibrational level would not be available for excitation.

For cold gases (T ≈ 300°K) the electron energy loss per collision with a homonuclear diatomic gas is independent of pressure and depends only on electron temperature. Since cross-modulation measurements of $G\nu$ are usually made in gases that are less than 0.1 per cent ionized, relatively few molecules are involved in inelastic encounters and there is always a plentiful supply of molecules which will not be in metastable levels. This implies that, in the case of cold (T ≈ 300°K) homonuclear diatomic gases, cross-modulation measurements of $G\nu$ for $T_e \approx 5000$°K would not depend upon whether $\tau_{EN} = 1/G\nu$ is less or greater than τ_{VIB}.

This analysis implies that whereas for a cold gas ($T = 300°K$, $T_e = 5000°K$) the measurements of $G\nu$ are dependent only on T_e, for a high temperature ($T = 5000°K$, $T_e = 5000°K$) homonuclear gas (N_2) the fractional energy loss of an electron depends both on electron temperature and whether $\tau_{EN} = 1/G\nu$ is much greater or much less than τ_{VIB}. For N_2 and O_2 the following relations should apply:

When $\tau_{EN} = 1/G\nu \ll \tau_{VIB}$ (High Pressure)

$$G(T = 5000°K, T_e = 5000°K) < G(T = 300°K, T_e = 5000°K)$$

When $\tau_{EN} = 1/G\nu \gg \tau_{VIB}$ (Low Pressure)

$$G(T = 5000°K, T_e = 5000°K) = G(T = 300°K, T_e = 5000°K) \quad .$$

For a diatomic molecule with a permanent dipole moment such as NO,

$$G(T = 5000°K, T_e = 5000°K) = G(T = 300°K, T_e = 5000°K)$$

regardless of the value of $G\nu$ ($\tau_{EN} = 1/G\nu$ is almost always longer than $\tau_{VIB}(NO) \approx 10^{-8}$ sec except at very high pressures).

Massey and Burhop[7] contend that the G values of O_2 are somewhat higher than the G values of N_2 for cold gases at $T_e \approx 0.5$ ev because of low-lying electronic states of the oxygen molecule which may be excited under electron impact. The low-lying electronic states of O_2 ($a\Delta_g$ at 0.8 ev and $b\,\Sigma_g^+$ at 1.3 ev) are metastable with enormous lifetimes of the order of 10 seconds. At $T = 3000°K$ (O_2 is fairly well dissociated at $T = 4000°K$ and $\rho/\rho_0 = 10^{-3}$), about 10 per cent of the O_2 molecules will be in the excited metastable low-lying electronic states. Only the rotational levels associated with the lowest vibrational state of the metastable electronic state of O_2 will be available for excitation under electron impact. Hence it is expected that at high gas temperatures ($T = 3000°K$), high pressures $\tau_{EN} = 1/G\nu \ll \tau_{VIB}$ and for $T_e \approx 0.5$ ev, the G values for O_2 will decrease from the cold gas G values corresponding to the same electron temperature. The interesting point is that the G values for O_2 should exhibit a more pronounced decrease than the G values for N_2 as the gas temperature is raised. The G values for NO should not show much of a change as the gas temperature is raised. A final point should be added about the effects of electron attachment on the measured G values of electronegative gases (NO and O_2). Although electron attachment to neutrals constitutes a loss of energy so far as the electron gas is concerned, the rates of electron attachment (three body) and detachment are so slow compared with the electron energy loss due to rotational, vibrational, and electronic excitation that electron attachment does not contribute to the relaxation time $\tau_{EN} = 1/G\nu$ as measured in a cross-modulation experiment. These interesting suggestions* lead to the possibility of a whole new host of experimental investigations of the G values of diatomic gases at high gas temperatures ($T \gtrsim 3000°K$).

It is hoped that the type of approach presented in this report will bring about a better understanding of the difficult problem of breakdown of missile antennas at high-power levels. The model of a plane-wave incident at arbitrary angle upon a nonlinear one-dimensional inhomogeneous multicomponent plasma slab may also be applicable to the interpretation of ionospheric radio-scattering data performed at high-power levels. Molmud[68] has already suggested using high-powered ground-based radio transmitters, such as the Puerto Rican Arecibo facility, to perturb the D region of the ionosphere. Induced changes in electron density and collision frequency may be observed by the changes in reflection coefficient they produce on the high-power transmitting wave or on a second wanted wave. From an examination of the shape of pulse returns, it may be possible to gain information on the dominant rate processes for electron attachment and recombination.

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A model is constructed of a nonlinear, one-dimensional, inhomogeneous, multicomponent plasma slab that has the characteristics of the plasma sheath surrounding a typical hypersonic re-entry vehicle at 200,000 ft while traveling at about 18,000 ft/second. A plane wave incident upon the slab at arbitrary angle can induce changes in electron temperature without affecting the neutral gas temperature. The variations of electron temperature produce changes in collision frequency and the rate coefficients describing the various electron production and loss mechanisms. Such changes cause alterations in the effective dielectric constant of the medium. On the basis of a model of a plane-layered medium composed of a stack of linear homogeneous sheets, the reflection and transmission coefficients of the nonlinear slab may be computed by a step-by-step

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